# Solving Electron Spin Drift-Diffusion Equations in Presence of Hyperfine Interactions



## INTRODUCTION

Spintronics is the study and exploitation of the spin of an electron and its associated magnetic moment.



Pure spin transport occurs when equal amounts of spin up and spin down electrons are travelling in opposite directions, producing a net charge current of zero.

A common method by which spin currents are generated is the Spin Hall Effect which relies on the spin-orbit interaction. There is also an Inverse Spin Hall Effect which entails the creation of an electrical current that is perpendicular to a spin polarized current.

In this work, we predict the generation of spin/charge currents from gradients in nuclear fields. The mechanism is akin to the famous Stern Gerlach effect.



based on File:Stern-Gerlach experiment.PNG of Theresa Knott

Large nuclear fields, induced by the hyperfine interaction, are known to influence spin transport characteristics in n-GaAs [1,2,3]. Nuclear fields are added to the spin drift-diffusion equation and the resulting spin distributions are calculated. Various boundary conditions are assumed in order to model various experimental arrangements Due to the complicated nature of the nuclear field, the steady state spin drift-diffusion equations are non-linear and must be solved numerically. In this work, we examine solutions for the spin distribution and spin current in the presence of a nuclear field. Lastly, the effect of magnetic field gradients on steady state spin are explored to show how these gradients affect spin distributions.

#### NUCLEAR FIELDS

The interaction between an electron and nuclear spin is in part described by the Fermi contact potential

 $U_{hf} \propto |\psi(\mathbf{R}_N)|^2 \mathbf{I} \cdot \mathbf{S}$ 

 $U_{hf} \propto \mathbf{B}_N \cdot \mathbf{S}$ 

The nuclear spin, in some ways, acts like a magnetic field

- there is a Zeeman-like interaction

- however the nuclear field does not generate a Lorentz force

Like in the Stern Gerlach effect, we expect that a gradient in nuclear field will separate spins

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Before incorporating nuclear field gradients, we first examined spin dynamics without nuclear fields present To model the spin dynamics, we set out to solve the spin drift diffusion equation in one dimension:

Analytic solutions were found for arbitrary field and any initial spin orientation at the origin

$$\mathbf{S} = e^{-z/L_{dr}} [(\hat{B}_0 \cdot \mathbf{S}_0) \hat{B}_0 e^{-|z|} \sqrt{(\frac{\mu E_z}{2D})^2 + \frac{1}{D\tau_s}} + (\mathbf{S}_0 - (\hat{B}_0 \cdot \mathbf{S}_0) \hat{B}_0 e^{-\frac{|z|}{L_s}} \cos \theta_0 + \frac{|z|}{L_s} \cos \theta_0 + \frac{1}{2} \sum_{k=1}^{N} \frac{1}{2k} \cos \theta_k + \frac{1}{2k} \sum_{k=1}^{N} \frac{1}{2k}$$

In the special case of magnetic field along z and the spin being along x at the origin, we recover results derived earlier

$$S_{x} = S_{0}e^{(\frac{-z}{L_{dr}} - \frac{|z|}{L_{s}})}\cos(\frac{z}{L_{0}})$$
$$S_{y} = S_{0}e^{(\frac{-z}{L_{dr}} - \frac{|z|}{L_{s}})}\sin(\frac{|z|}{L_{0}})$$

These results are plotted on the right.

The spin drift diffusion equations can also be solved, in the steady state, when a nuclear field is present. The solutions are numerical in that case.

We derive new spin and charge drift diffusion equations when there is a generic magnetic field gradient.

$$\begin{pmatrix} \frac{ds}{dt} \end{pmatrix} = D\nabla^2 s - \left(\frac{q\tau}{m}\right) s\nabla * E - \left(\frac{q\tau}{m}\right) (\nabla s) * E - \left(\frac{\gamma\hbar\tau}{2m}\right) [(\nabla n) * \nabla B_z + n\nabla^2 B_z + n\nabla^2 B_z + \frac{q\tau}{dt}] = D\nabla^2 n - \left(\frac{q\tau}{m}\right) n\nabla * E - \left(\frac{q\tau}{m}\right) (\nabla n) * E - \left(\frac{\gamma\hbar\tau}{2m}\right) [(\nabla s) * \nabla B_z + s\nabla^2 B_z + \frac{q\tau}{dt}] = D\nabla^2 n - \left(\frac{q\tau}{m}\right) n\nabla * E - \left(\frac{q\tau}{m}\right) (\nabla n) * E - \left(\frac{\gamma\hbar\tau}{2m}\right) [(\nabla s) * \nabla B_z + s\nabla^2 B_z + \frac{q\tau}{dt}] = D\nabla^2 n - \left(\frac{q\tau}{m}\right) n\nabla * E - \left(\frac{q\tau}{m}\right) (\nabla n) + E - \left(\frac{q\tau}{2m}\right) [(\nabla s) + \nabla B_z + \frac{q\tau}{dt}] = D\nabla^2 n - \left(\frac{q\tau}{m}\right) n\nabla + E - \left(\frac{q\tau}{m}\right) (\nabla n) + E - \left(\frac{q\tau}{2m}\right) [(\nabla s) + \nabla B_z + \frac{q\tau}{dt}] = D\nabla^2 n - \left(\frac{q\tau}{m}\right) n\nabla + E - \left(\frac{q\tau}{m}\right) (\nabla n) + E - \left(\frac{q\tau}{2m}\right) [(\nabla s) + \nabla B_z + \frac{q\tau}{dt}] = D\nabla^2 n - \left(\frac{q\tau}{m}\right) n\nabla + E - \left(\frac{q\tau}{m}\right) (\nabla n) + E - \left(\frac{q\tau}{2m}\right) [(\nabla s) + \nabla B_z + \frac{q\tau}{dt}] = D\nabla^2 n - \left(\frac{q\tau}{m}\right) n\nabla + E - \left(\frac{q\tau}{m}\right) (\nabla n) + E - \left(\frac{q\tau}{2m}\right) [(\nabla s) + \nabla B_z + \frac{q\tau}{dt}] = D\nabla^2 n - \left(\frac{q\tau}{m}\right) n\nabla + E - \left(\frac{q\tau}{m}\right) (\nabla n) + E - \left(\frac{q\tau}{2m}\right) [(\nabla s) + \nabla B_z + \frac{q\tau}{dt}] = D\nabla^2 n - \left(\frac{q\tau}{m}\right) n\nabla + E - \left(\frac{q\tau}{m}\right) (\nabla n) + E - \left(\frac{q\tau}{dt}\right) = D\nabla^2 n - \left(\frac{q\tau}{m}\right) n\nabla + E - \left(\frac{q\tau}{m}\right) (\nabla n) + E - \left(\frac{q\tau}{dt}\right) = D\nabla^2 n - \left(\frac{q\tau}{m}\right) n\nabla + E - \left(\frac{q\tau}{m}\right) (\nabla n) + E - \left(\frac{q\tau}{dt}\right) = D\nabla^2 n - \left(\frac{q\tau}{m}\right) + E - \left(\frac{q\tau}{m}\right) + E - \left(\frac{q\tau}{dt}\right) = D\nabla^2 n - \left(\frac{q\tau}{m}\right) + E - \left(\frac{q\tau}{m}\right) + E - \left(\frac{q\tau}{dt}\right) = E - \left(\frac{q\tau}{dt}\right)$$

We have generalized spin and charge drift diffusion equations to account for magnetic fields gradient. Our results show that up and down spins separate when there is an initial charge impulse. When there is an initial spin impulse, non-uniform charge is induced.

[1] M. Chan et al. Phys. Rev. B 80, 161206, (2009) [2] N. J. Harmon, T. A. Peterson, C. C. Geppert, S. J. Patel, C. J. Palmstrøm, P. A. Crowell, and M. E. Flatté, Phys. Rev. B 92, 140201(R) (2015). [3] Y.-S. Ou, Y.-H. Chiu, N. J. Harmon, P. Odenthal, M. Sheffield, M. Chilcote, R. K. Kawakami, and M. E. Flatté, Phys. Rev. Lett. **116**, 107201 (2016).

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### SPIN DRIFT DIFFUSION EQUATIONS

$$D\frac{\partial \boldsymbol{S}(z,t)^2}{\partial z^2} + \mu E_z \frac{\partial \boldsymbol{S}(z,t)}{\partial z} + \gamma \boldsymbol{B}_0 \times \boldsymbol{S}(z,t) - \frac{1}{\tau_s} \boldsymbol{S}(z,t)$$

 $P_x$ ,  $P_y$ 

-20

-30

### CONCLUSION

### REFERENCES









=0.

In one dimension, with magnetic field and gradient both along the single spatial dimension, we find

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