[Bridges: A Journal of Student Research](https://digitalcommons.coastal.edu/bridges)

Issue 13

Article 5

2020

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ABSTRACT

Sports have developed over time into an entire industry from the children who grow up playing sports to the professional players that fans love to watch. Because all sports involve movements of the human body, a bio-mechanical system, these movements may be modeled with mathematics and physics. Specifically, the sports of golf, tennis, and baseball can use the concepts of a double pendulum to maximize sport performance. The double pendulum model is applied to these sports by making the arm one pendulum and the club, racquet, or bat the second pendulum. A good understanding of the mechanics of a double pendulum and its connection to these sports can enhance the efforts of players and aid them in their performance.

I. INTRODUCTION

A pendulum is defined as a small mass that is suspended from a wire or string that allows the mass to swing back and forth in a semi-circular motion [1]. A double pendulum is a similar system, but with two pendulums attached at a "joint". The double pendulum is a mathematical system that can be represented by two differential equations derived from Newton's Second Law of Motion, $F = ma$. Appendix A provides the derivation of a single pendulum to illustrate the origins of the differential equations representing a double pendulum. A visual of a double pendulum can be seen in figure 1.

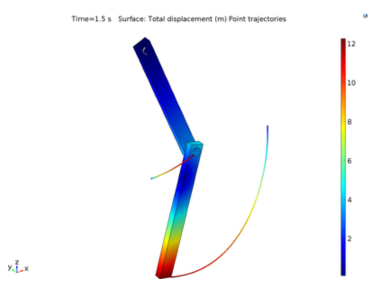

FIG. 1. Visual Representation of a Double Pendulum [2]

A. Introduction to Double Pendulum Model for Golf

Golf was the original sport to be modeled by a double pendulum. For this model, it is assumed that the arms are one pendulum and the club is the other pendulum with the wrist being the pivot point. It is assumed that the wrist 'hinge' is restricted to prevent the club segment from going too far behind the golfer's head [3]. This is a combination of the tennis and baseball models.

B. Introduction to Double Pendulum Tennis Model

We will next apply the general knowledge of a double pendulum motion to the sport of tennis. Here, the forearm is one pendulum and the racquet is the second pendulum, with the joint or connection point being the person's wrist. Looking then at the physics of a tennis stroke, one can observe that gravity plays a minor role because most of the forces in the system stem from the muscles in the arm and hand that swing in a horizontal or vertical plane [4]. Thus, analyzing this model will yield results that tennis players can use to potentially maximize their strokes and serves.

C. Introduction to Double Pendulum Baseball Model

A double pendulum can model two aspects of baseball. The first instance is hitting the ball with a bat. This case differs from the tennis model in that there are two hands on the bat instead of one, and so there are more body segments to consider. As a result, the mechanics of swinging a bat appears to be more complex modeling problem. One way to simplify the analysis of such mechanics is to view both arms acting together as a single pendulum segment hinged to the bat at the wrist [5]. The bat would remain the second pendulum. This is a simpler way to look at the system of swinging a bat to get a better understanding of what is happening mechanically.

The second aspect of baseball that can be modeled by a double pendulum is pitching. This system consists of the arm being one pendulum, while the hand is the second pendulum. The wrist acts as the pivot or joint connection [5].

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II. ANALYZING DOUBLE PENDULUM SWING EXPERIMENT

Again, the sports of tennis, baseball, and golf all have in common a swing of some sort. Whether that is swinging a racquet, club, or bat, the double pendulum is modeling this motion to get a desired result in the specific sport. This result can vary from maximizing the velocity of a tennis serve or stroke to maximizing the trajectory of a baseball or golf ball. With this in mind, double pendulum experiments have in fact been conducted to try to determine what a "perfect swing" might look like [6].

One paper described an experiment that constructed a double pendulum out of two rods to mimic the swinging motion done by athletes in the sports of tennis, golf, or baseball. Essentially, the experiment used a video recording to break down the swinging motion of a double pendulum to determine a way to transfer all the kinetic energy from the swing to the ball. The idea was to find a way to achieve the maximum transfer for each of the subject sports [6]. This experiment defined a "perfect swing" as the transfer of all initial potential energy to kinetic energy in the swing of the rod to all that kinetic energy being transferred into the ball [6].

A. Conclusion of Swing Experiment

The results of the preceding experiment presented in literature revealed a couple of key characteristics of a "perfect swing" [6]. Firstly, the arm and the instrument should be lined up fairly closely. Secondly, the goal is to have the angular momentum of the instrument reach its maximum value at the moment of impact with the ball [6].

Based on the results of the swing experiment, it is important to note what was discovered by these researchers [6]. Essentially, the experiment was using a self-made double pendulum to see how the motion could correlate to a tennis, baseball, or golf swing. This highlighted a way that physics and mathematics could be correlated to sports enhancement. The experiment demonstrated that analyzing a double pendulum can help to determine the best points of contact in a swing which yields the most desired results. This means that the experiment found the most ideal position to swing at a ball based on the sport. Therefore, this is showing how a double pendulum can be used to model sports and maximize the results.

III. DOUBLE PENDULUM MODEL MAXIMIZING GOLF, TENNIS, AND BASEBALL

Thus far, models have been established to show that golf, tennis, and baseball, can be modeled by a double pendulum. How is that really useful though? Simply stating that these sports can be modeled by a double pendulum does not show why that concept is useful or allow for mathematics and physics to be correlated to the world of sports. To better understand why this is important, the described models were tested to determine ways of maximizing certain results.

A. Double Pendulum Model Maximizing Golf Swing

The model of a double pendulum for a golf swing was used to study the mechanics of a golf swing and contributed to the sport with a new optimal way to swing the club. The model itself is fairly simple and looks specifically at the downswing. The results were that the combination of the effect from inertia and centripetal force acting on the club can create an efficient downswing if the arms are accelerating using the correct force [3]. This method eliminates wrist torque which led to a concept in golf known as the natural wrist release [3]. Eliminating wrist torque is essentially helping golfers by allowing the club to accelerate naturally towards the desired goal of hitting the ball. This means that based on the model, researchers have experimented and found a way to make a golf swing more efficient, which in turn maximizes the sport of golf because a better swing increases trajectory of the ball.

B. Double Pendulum Maximizing Tennis

As previously stated, the model for a double pendulum for tennis correlates to one pendulum being the arm and the second pendulum being the racquet with the wrist the joint [4]. Figure 2 illustrates this relationship[7].

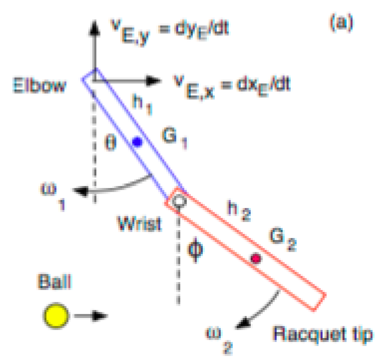

FIG. 2. Double Pendulum Model for Tennis [4]

Once again, this model is effective in displaying the system but to utilize it, is to look at how this model can maximize some aspect of the sport of tennis. The model for tennis can be maximized by looking at the best way to collide with the ball at a given stroke for a desired return[7]. The swing experiment of section 2 discusses the transfer of energy to get the maximum output of the ball and the same can be discussed here. A way to generate a better angular velocity is through the energy transfer of one pendulum to the next or from the arm to the racquet[7]. To best achieve this, the angular velocity of the arm portion should be zero to allow the angular velocity of the racquet to reach its maximum value [7]. Another observation is that the speed of the rebound ball can increase by 20 percent depending on the torque. Therefore, the sport of tennis can not only be modeled by a double pendulum but can be analyzed using said pendulum to get results that can lead to an efficient way to hit a tennis ball.

C. Double Pendulum Model Maximization of Baseball

Baseball, in itself, is a unique sport because it is a very mechanical sport that can use different aspects of physics to enhance the players motions for a better outcome. For example, pitchers focus on small mechanical adjustments to increase velocity. The introduction explained the mechanics involved with a baseball swing and how a double pendulum can model this. It also introduced another aspect of baseball which can model a double pendulum that is pitching. The double pendulum is useful in understanding the mechanics of a baseball swing[5] but looking at a way to maximize the sport, leads to a focus on the pitching model. Figure 3 shows the comparison of the definition of a double pendulum to the model applied to a pitching arm [8].

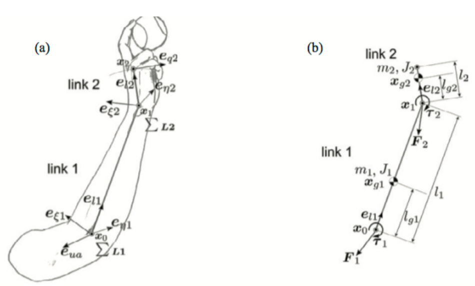

FIG. 3. The figure on the left, a, shows the model of a baseball pitch while the figure of the right, b, is a simple double pendulum model [8].

A study was done to determine the best way to produce maximum velocity by looking at how non-muscular forces like gravity forces generate, absorb, and transfer mechanical energy [8]. The study was modeled by a three-dimensional double pendulum with a moving pivot model to use a multi-body power analysis [8]. This means that the a double pendulum model was derived and then used in this study to analyze the dynamic behavior associated with maximizing a pitch. The focus of the study was the assessment of the motion of the forearm and wrist during a pitch. The experiment associated with this study consisted of using video footage to record a collegiate male baseball pitcher. The results yielded that the hand segment transferred mechanical energy from the internal force which consisted primarily of the centrifugal force [8]. Another observation was that the muscle torque applied to the wrist joint didn't increase the energy but instead, absorbed the mechanical energy from the hand segment. Thus, the wrist is not able to create a "high ball velocity"[8]. Therefore, this article used a real life pitcher and the double pendulum model to see the points at which a pitcher could achieve a maximum velocity by analyzing the transfer of mechanical energy in the system. This uses a double pendulum model to maximize an aspect of a sport.

IV. CONCLUSION

Baseball, golf, and tennis are three sports that have no real player to player physical contact. Other sports have the added unpredictable variables of one on one contact within each play. This makes them unique systems that can be modeled by other physics or mathematical methods. For the purpose of this paper, they were all modeled to a double pendulum. The purpose of modeling them was to find a way to use the physics of a double pendulum to improve the sport in some way. Golf used the model to discover a back swing, and to better understand the mechanics of the swing itself. Tennis used the model to find an efficient way to hit the ball. Lastly, baseball used the model to understand the mechanics of a baseball swing and to maximize the points at which a pitcher could achieve maximum velocity. Thus, a double pendulum is a simple mathematical system but when applied to the world of sports, it can yield helpful results.

Appendix A: Derivation of a Single Pendulum

A way of understanding the differential equations of a double pendulum is to first analyze a single pendulum and how it is derived. The first step is to look at a free body diagram and see what forces are acting on the joint where the mass is hung. Thus, figure 4 represents the free body diagram of a single pendulum[1].

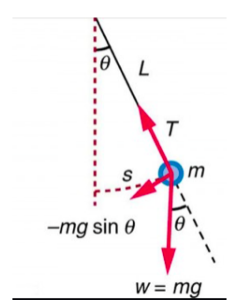

FIG. 4. Free body diagram of a single pendulum [1]

Thus, from the free body diagram an equation can be formed using Newton's second law, $F = ma$. To find the total force, F, the individual forces on the free body diagram must be added as follows,

$$F = ma$$
$$T\cos\theta j - T\sin\theta i - mgj = ma$$

Now, we have just found the left side of the equation, the sum of the forces. To find the right side, we will look at what the acceleration will be and multiply that by the mass. In physics, acceleration is the second derivative of the position and velocity is the first derivative of position which is found using kinematics. Kinematics is defined as the motion of objects without looking at forces. For this, trigonometry is used to find the initial positions in the x and y direction. Thus, the position of the pendulum is,

$$s(t) = L\sin\theta i - \cos\theta j$$
$$v(t) = \theta' L\cos\theta i + \theta' L\sin\theta j$$
$$a(t) = L[\theta''\cos\theta i - \theta'^2\sin\theta i + \theta''\sin\theta j + \theta'^2\cos\theta j].$$

The above equations are simply showing the derivative of the position to get velocity and then the derivative of velocity to get the acceleration. Therefore, to finish Newton's second law, $F = ma$, the mass just needs to be multiplied by the acceleration found above and then set equal to the sum of the forces.

$$T\cos\theta j - T\sin\theta i - mgj$$
$$= m(L[\theta''\cos\theta i - \theta'^2\sin\theta i + \theta''\sin\theta j + \theta'^2\cos\theta j])$$

To simplify that equation, the terms are separated into two equations by the unit vectors. Thus,

$$i : -T\sin\theta = mL(\theta''\cos\theta - \theta'^2\sin\theta) \tag{A1}$$

$$j : T\cos\theta - mg = mL(\theta''\sin\theta + \theta'^2\cos\theta). \tag{A2}$$

Now, we will eliminate the T terms by multiplying equation A1 by $\cos\theta$ and equation A2 by $\sin\theta$ to get

$$-T\sin\theta\cos\theta = mL(\theta''\cos^2\theta - \theta'^2\sin\theta\cos\theta) \tag{A3}$$

$$T\sin\theta\cos\theta - mg\sin\theta = mL(\theta''\sin^2\theta + \theta'^2\cos\theta\sin\theta) \tag{A4}$$

Using algebraic manipulation and simplification we get,

$$\theta'' = -\frac{g}{L}\sin\theta \tag{A5}$$

The above is the second order differential equation for a single pendulum.

A double pendulum follows the same method but instead of one differential equation it is derived into two second order differential equations because there are four initial equations from the free body diagram and the forces involved. The free body diagram for a double pendulum is displayed in figure 5.

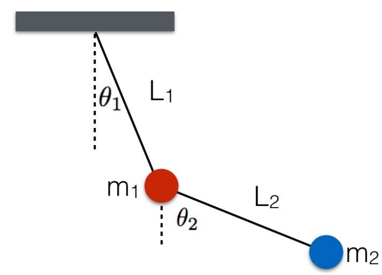

FIG. 5. Free body diagram of a Double Pendulum[9]

Following the same steps as above, the differential equations depicting a double pendulum are as follows,

$$\theta_1'' = \frac{-g(2m_1 + m_2)\sin\theta_1 - m_2 g\sin(\theta_1 - 2\theta_2) - 2\sin(\theta_1 - \theta_2)m_2(\theta_2'^2 L_2 + \theta_1'^2\cos(\theta_1 - \theta_2))}{L_1(2m_1 + m_2 - m_2\cos(2\theta_1 - 2\theta_2))}$$
$$\theta_2'' = \frac{2\sin(\theta_1 - \theta_2)(\theta_1'^2 L_1(m_1 + m_2) + g(m_1 + m_2)\cos\theta_1 + \theta_2'^2 L_2 m_2\cos(\theta_1 - \theta_2)}{L_2(2m_1 + m_2 - m_2\cos(2\theta_1 - 2\theta_2))}$$

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