8-8-2017

The Application of Proper Orthogonal Decomposition to Numerically Modeled and Measured Ocean Surface Wave Fields Remotely Sensed by Radar

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THE APPLICATION OF PROPER ORTHOGONAL DECOMPOSITION TO
NUMERICALLY MODELED AND MEASURED OCEAN SURFACE WAVE FIELDS
REMOTELY SENSED BY RADAR

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Submitted in Partial Fulfillment of the
Requirements for the Degree of Master of Science in
Coastal Marine and Wetland Studies in the
Department of Coastal and Marine Systems Science
School of the Coastal Environment
Coastal Carolina University
Summer 2017

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Acknowledgements

I would like to thank my advisor, Dr. Erin Hackett, for her invaluable direction, support, and patience throughout this project. Without her instrumental expertise and guidance this thesis would not have been possible. I would also like to thank my thesis advisory committee, Dr. Roi Gurka and Dr. Diane Fribance, for the valuable input and knowledge. I would like to acknowledge Dr. Gordon Farquharson (The Applied Physics Lab of the University of Washington), Tony de Paolo (Scripps Institute of Oceanography), Dr. Joel Johnson and Shanka Wijesundara (Ohio State University), Dr. Craig Merrill (Naval Surface Warfare Center; Carderock Division), and Dr. Kevin Cockrell (Applied Physical Sciences) for the provision of data used in this study. I would also like to thank the Office of Naval Research for the funding to conduct this study (Grant N00014-15-1-2044, Dr. Paul Hess III) as well as the Coastal Carolina University Research Council (Grant 17-4862) for the opportunity to present this work at a major conference. I am appreciative of the assistance and support of my peers, namely Josh Humberston, Kolton Cooper, and Meghan Troup, for their work in buoy data processing, as well as all my peers in the Environmental Fluids Lab and DCMSS. Finally, I am grateful to my family for their unwavering support and encouragement.
Abstract

Phase-resolved ocean surface wave elevation maps provide important information for many scientific research areas (e.g., rogue waves, wave-current interactions, and wave evolution/growth) as well as for commercial and defense applications (e.g., naval and shipping operations). To produce these maps, measurements in both time and space are necessary. While conventional wave sensing techniques are limited spatially, marine radar has proven to be a complex yet promising remote sensing tool capable of providing both temporal and spatial wave measurements. The radar return from the sea surface is complex because it contains contributions from many sources only part of which provide information about the ocean surface wave field. Most existing techniques used to extract ocean wave fields from radar measurements implement fast Fourier transforms (FFTs) and filter this energy spectrum using the linear dispersion relationship for ocean waves to remove non-wave field contributions to the radar signal. Inverse Fourier transforms (IFFTs) return the filtered spectrum to the spatial and temporal domain. However, nonlinear wave interactions can account for a non-negligible portion of ocean wave field energy (particularly in high sea states), which does not completely adhere to the linear dispersion relationship. Thus, some nonlinear wave energy is lost using these FFT dispersion-filtering techniques, which leads to inaccuracies in phase-resolved ocean surface wave field maps. This deficiency is significant because many of the aforementioned research areas and applications are most concerned with measurement and prediction of such anomalous wave conditions.

Proper orthogonal decomposition (POD) is an empirical technique used in scientific fields such as fluid mechanics, image processing, and oceanography (Sirovich, 1987). This
technique separates a signal into a series of basis functions, or modes, and time or spatial
series coefficients. Combining a subset of the modes and coefficients can produce a
reduced order representation of the measured signal; this process is referred to as a
reconstruction. This research applies POD to radar Doppler velocity measurements of the
sea surface and uses the leading modes as a filter to separate wave contributions to the
radar measurement from non-wave contributions. In order to evaluate the robustness of this
method, POD is applied to ocean wave radar measurements obtained using three different
radar systems as well as to numerically modeled radar data for a variety of environmental
conditions. Due to the empirical nature of the POD method, the basis functions have no
innate physical significance, therefore the shape and content of leading POD modes is
examined to evaluate the linkage between the mode functions and the wave field physics.
POD reconstructions and FFT-based methods are used to compute wave field statistics that
are compared with each other as well as to ground truth buoy measurements. Correlation
coefficients and root mean squared error are used to evaluate phase-resolved wave orbital
velocity time series reconstructions from POD and FFT-based methods relative to ground
truth buoy velocity time series measurements. Results of this study show that when POD
is applied to radar measurements of the sea surface: (i) the leading mode basis functions
are oscillatory and linked to the physics of the measured wave field; (ii) POD performs
comparably to FFT-based dispersion filtering methods when calculating wave statistics;
and (iii) phase-resolved POD orbital velocity maps show higher correlations with buoy
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List of Symbols and Acronyms

$A$: Antenna

$a$: Wave amplitude

ADCP: Acoustic Doppler current profiler

AWAC: Acoustic wave and current meter

$B$: POD matrix

$b$: Time dependent constant

$C$: Correlation-coefficient

$c_g$: Group velocity

$D$: Doppler velocity

$D_\theta$: Direction distribution

$d_k$: Dispersion filter

$E_{rms}$: Root mean squared error

EM: Electromagnetic

EOF: Empirical orthogonal function analysis

$f$: Linear frequency

$f_c$: Center frequency

$f_{ny}$: Nyquist frequency

FFT: Fast Fourier transform

$g$: Acceleration due to gravity

$H$: Wave height

$h$: Water depth

$H_{ci}$: Significant wave height confidence intervals
$H_s$: Significant wave height

$HH$: Horizontal transmit and horizontal receive polarization

$k$: Radian wavenumber

$k_x$: Radian wavenumber in $x$-direction

$k_y$: Radian wavenumber in $y$-direction

$LMSR$: Large, Medium-Speed, Roll-on/Roll-off

$M$: Number of points in data series

$m_0$: Zeroth spectral moment

$MLP$: Mobile Landing Platform

$N$: Number of radar frames

$n$: Number of POD modes

$P$: POD mode functions

$P_o$: Radar Doppler velocity spectrum energy

$p_k^T$: POD basis functions

$P_r$: Reconstructed velocity spectrum energy

PCA: Principal component analysis

PSD: Power spectral density

POD: Proper orthogonal decomposition

PRF: Pulse repetition frequency

$Q$: POD matrix

$q_k$: Spatial or temporal POD coefficients

$r$: Range spatial coordinate

RMSE: Root mean squared error
\( S(f) \): Linear frequency spectrum

\( S(k) \): Radian wavenumber spectrum

\( S(\omega) \): Radian frequency spectrum

\( S_h \): Sea surface elevation spectrum

\( S_v \): Orbital velocity spectrum

\( s_b \): Standard deviation of buoy measured orbital velocity time series

\( s_e \): Standard deviation of estimated orbital velocity time series

SVD: Singular value decomposition

SWL: Still water level

\( T \): Mean wave period

\( t \): Time

\( T_{DF} \): Total degrees of freedom

\( T_e \): Percentage of reconstructed energy

\( T_p \): Peak wave period

\( U \): Current velocity

\( U_w \): Wind Speed

\( u \): Horizontal velocity component

USNS: United States Naval Ship

\( v \): Horizontal velocity component

\( V_b \): Buoy measured orbital velocity

\( \tilde{V}_b \): Mean buoy measured orbital velocity

\( V_e \): Estimated orbital velocity

\( \bar{V}_e \): Mean estimated orbital velocity
\( V_{rms} \): Root mean square velocity

\( VV \): Vertical transmit and vertical receive polarization

\( W \): Dispersion filter width

\( w \): Vertical velocity component

\( x \): Horizontal spatial coordinate

\( y \): Horizontal spatial coordinate

\( z \): Vertical spatial coordinate

\( \alpha \): Significance level

\( \Delta f \): Linear frequency resolution

\( \Delta f_b \): Bandwidth

\( \Delta t \): Sample interval

\( \Delta \Theta \): Angle between wave systems

\( \Delta \omega \): Radian frequency resolution

\( \Sigma \): Singular value matrix

\( \eta \): Sea surface elevation

\( \sigma \): Radian frequency from dispersion relationship including current

\( \Theta \): Wave propagation direction

\( \Theta_p \): Peak wave direction

\( \Theta_r \): Radar look direction

\( \Theta_w \): Wind direction

\( \phi \): Radar azimuth

\( \Psi \): Velocity potential

\( \omega \): Radian frequency
\( \chi^2 \): Chi squared probability

\( \lambda \): Wavelength

\( \lambda_p \): Peak wavelength

1D: One dimensional

2D: Two dimensional

3D: Three dimensional
1.0. Introduction

Use of radar as a remote sensor for measuring the marine environment has been a topic of interest since the 1960s (Ijima et al., 1964; Wright, 1965; Wright, 1966). Remote sensors provide advantages over conventional in situ sensors. For example, in situ sensors only provide a measurement at one point in space whereas remote sensors provide broad spatial coverage. Use of remote sensors can also provide a financial advantage because the majority of the cost in making in situ measurements is derived from the deployment, recovery, and maintenance of the sensor, which is reduced to only maintenance for a remote sensor. Conversely, remote sensors are often less accurate than in situ sensors because a number of external factors affect the measurement.

Marine radar has become a prominent tool for remote sensing of ocean surface waves (Holman and Haller, 2013). Radar returns from the sea surface are generally referred to as “sea clutter.” The extraction of wave parameters from radar measurements is complex due to numerous factors that affect the measurement, such as radar system properties, ocean currents, wind speed (surface roughness), wave height, and wave steepness. The radar signal can also be impacted by anything in the path of the electromagnetic (EM) radar wave either on the ocean surface or in the air such as boats, buoys, birds, rain, etc. Parameters of the radar system such as pulse length, bandwidth, center frequency, polarization, and antenna geometry also impact the radar measurements, as well as EM interference from other radars or EM systems.
Recently there has been much interest in deriving wave statistics and accurate phase-resolved wave fields from radar measurements (Holman and Haller, 2013; Carrasco et al., 2017; Lyzenga, 2017). Naval and shipping applications as well as oceanographic research areas would benefit from the ability to produce accurate phase-resolved ocean surface wave elevation maps covering a relatively large spatial area (Alford et al., 2016), as well as statistical properties of the wave field (Carrasco et al., 2017). Such real-time ocean surface maps would increase safety and operational awareness for commercial and defense applications, and aid in furthering knowledge in oceanographic research areas such as rogue waves, wave evolution/growth, and wave/current interactions.

Conventionally, wave field statistics are extracted from radar measured sea clutter using fast Fourier transforms (FFTs) of the backscatter intensity, where the spectrum is filtered based on the linear dispersion relationship for ocean surface waves in order to remove non-wave contributions to the signal as described in the seminal paper of Young et al. (1985). Similar techniques can be applied to Doppler velocities measured with coherent radar systems. However, it is known that in many cases non-negligible parts of ocean wave fields are nonlinear, and thus, this portion of the wave field is eliminated using the dispersion relationship filtering techniques (Smith et al., 1996). Wavenumber-frequency spectra of the ocean surface typically have a linear energy feature at frequencies lower than that associated with the dispersion relationship (see Figure 13, panels A and C, section 3 for an example). This feature is referred to as the group line and is most likely related to wave breaking caused by interactions of waves of different wavelengths (Pla and Farquharson, 2012). By filtering out such features, the accuracy of phase-resolved ocean surface elevation maps may be limited.
An alternative method for computing phased resolved ocean wave fields from sea clutter is proper orthogonal decomposition (POD). POD is also referred to as empirical orthogonal function analysis (EOF), principal component analysis (PCA), or singular value decomposition (SVD), and has been applied extensively to nonlinear phenomena such as turbulence (Holmes et al., 1997) and other oceanographic applications (Fullerton and Punzi, 2016). POD is a technique used to provide low-dimensional representations of complex, high-dimensional systems. The POD method decomposes a signal into a set of modes (or basis functions) and time series or spatial series coefficients. The measured signal can be reconstructed by the summation of the modes multiplied by the corresponding coefficients. A reduced order representation of the signal can be created using a sub-set of the modes; this is referred to as a reconstruction. This technique makes no assumption of the basis function’s form a priori; instead, the basis functions are determined by the data. This feature differentiates POD from FFT methods in that FFT methods assume sinusoidal basis functions. The determination of the basis functions a posteriori minimizes the number of POD modes required to account for the majority of the variance of the signal, which is the premise upon which the basis functions are determined. While there is no inherent physical interpretation to the mode basis functions, there is an increasing body of work associating POD basis functions with physical significance (e.g. Kerschen and Golinval, 2002; Diamessis et al., 2010; Hackett et al., 2014).

This method can be used to reconstruct phase-resolved ocean wave fields from radar measured sea clutter using the leading mode functions as a filter to separate wave contributions to the radar signal from non-wave contributions, assuming some of the basis functions can be associated with the physics of the ocean surface waves while others can
be associated with unwanted artifacts of the radar measurement. If feasible, then the need to filter data based on the linear dispersion relationship is removed, potentially allowing features such as the group line to be included in the retrieved wave fields. The method also does not require entering the spectral domain, thus reducing sampling requirements because the spectral resolution is determined by the duration of the dataset. POD could also reduce data storage needs because the data could be represented and reconstructed with less information than contained in the original measurement. POD techniques are also less computationally intensive than FFT based methods, which would be an advantage in real-time computation of ocean surface wave fields.

It has been shown that when POD is applied to modeled ocean wave fields of various complexity, POD modes can be connected to the physics of the wave field (Hackett et al., 2014). The goals of this study are (i) to determine if this physical basis to the POD mode functions holds when the method is applied to radar measurements of the sea surface, (ii) to compare wave field statistics computed from POD reconstructions to those computed from conventional FFT-based dispersion filtering methods, and (iii) to compare accuracy of phase-resolved wave fields computed from these methods. The objectives are achieved through analysis of several radar datasets collected over a range of environmental conditions as well as using simulations of radar returns from the sea surface. For objectives ii and iii, results are also compared with conventional ground truth sensors, i.e., wave buoys, or wave field information used to initialize the simulated radar returns.

The following section will provide a brief background on ocean surface waves, conventional wave measurement techniques, and radar wave measurement techniques. The third section discusses data used in this study. The fourth section details the analysis
methods. The results of the study are discussed in the fifth section. The last section of the thesis is a summary and conclusions chapter.
2.0. Background

This section provides background information on the physics of ocean surface waves, conventional methods of measuring ocean surface waves, as well as measuring the ocean surface using radar remote sensing, including scattering of EM radar waves and methods of processing radar measurements of the sea surface.

2.1. Ocean Surface Waves

Ocean surface waves are generated by forces acting to displace the water surface. The magnitude, length and temporal scale of the displacement determine the scales of the waves generated in response. Ocean surface waves exist at scales from less than a centimeter to hundreds of kilometers depending on the generating mechanism. Wind generates ocean waves of scales from small capillary waves (centimeter scale), to swell (hundreds of meters) by exerting stress on the sea surface. The longest ocean waves, in the form of tides, are generated by the gravitational forces between the sun and moon, and the Earth. These waves have wavelengths of up to half the circumference of the globe.

2.1.1. Linear Wave Theory

Because wind driven ocean waves (approximately 2-30 seconds in period) are of significant impact to naval and many oceanographic applications, this study focuses on waves of this scale. Wind imposes a stress on the ocean surface and once the disturbance is generated, gravity and pressure cause the waves to propagate away from their source.
Figure 1 (adapted from Dean and Dalrymple, 1991) illustrates standard parameters used in linear wave theory to describe and characterize ocean surface waves. Wavelength ($\lambda$) refers to the distance from one crest to the next of a single wave, while period, $T$, refers to the time it takes for a wave of a single wavelength to pass a fixed point (i.e., the time from one crest to the next). Wave height ($H$) refers to the distance vertically from the trough to the crest of a wave, while amplitude ($a$) refers to the vertical distance from the still (or mean) water level (SWL) to the crest of a wave (i.e. $a = \frac{1}{2}H$), water depth ($h$) is the distance from the SWL to the sea floor. The vertical displacement of the wave from the SWL at any given position and time is referred to as the free surface elevation ($\eta$).

A wave field is often characterized using a power spectrum of the surface elevation. The power spectral density (PSD) is calculated through a fast Fourier transform (FFT) of sea surface height ($\eta$), and shows the amount of energy with respect to frequency. The low and high frequency limits of the spectrum as well as the resolution of the spectrum depend on the duration and frequency of sampling. The lowest frequency able to be resolved, as well as the frequency resolution ($\Delta f$), is inversely dependent on the length of the sampling period:

$$\Delta f = \frac{1}{M\Delta t}$$

(1)

where $\Delta t$ is the sampling rate of the data and $M$ is the number of points in the data series. The maximum frequency able to be resolved, referred to as the Nyquist frequency ($f_{ny}$), is dependent on the sampling frequency, and is equal to:
\[ f_{ny} = \frac{1}{2\Delta t} \]  

Figure 2 shows a qualitative schematic of a wave power spectrum with various periods and forcing mechanisms for each band of wave energy. As previously mentioned, this research will focus on wind driven gravity waves. Wave statistics such as peak period \( T_p \) - the period associated with the highest wave energy), peak wavelength \( \lambda_p \), mean period \( T \), and significant wave height \( H_s \) - the height of the highest one-third of the waves), can be computed using the wave height power density spectrum (Dean and Dalrymple, 1991; Earle, 1996).

Short period waves generated by local winds are typically referred to as wind waves or wind seas, and have periods of approximately 2 to 6 seconds. Waves that originated (due to a wind event) at a remote location are referred to as swell, and have periods ranging from approximately 7 to 20 seconds. Figure 3 shows a conceptual representation of swell waves versus wind waves (adapted from Dean and Dalrymple, 1991).

Linear wave theory is a linearized description of the propagation of gravity waves on the surface of a homogeneous fluid layer, assuming uniform mean depth, inviscid, incompressible and irrotational fluid. These assumptions result in a second order differential equation, the Laplace equation:

\[ \nabla^2 \psi = 0 \]
where $\Psi$ is the velocity potential, which is related to the velocity components ($u$, $v$, and $w$) by:

\begin{align*}
    u &= -\frac{\partial \Psi}{\partial x} \quad (4) \\
    v &= -\frac{\partial \Psi}{\partial y} \quad (5) \\
    w &= -\frac{\partial \Psi}{\partial z} \quad (6)
\end{align*}

where $x$, $y$, and $z$ are spatial coordinates. To solve equation 3 in 2D ($x$ and $z$) to obtain the velocity potential function, four boundary conditions are applied. The first boundary condition is the kinematic free surface boundary condition, which states that the fluid particles on the water surface must be free to follow the vertical motion of the wave:

\begin{equation}
    w = \frac{\partial \eta}{\partial t} + u \frac{\partial \eta}{\partial x} \quad \text{on } z = \eta(x, t) \quad (7)
\end{equation}

where $t$ is time. Note that the coordinate system is defined as shown in Figure 1. The second boundary condition is the dynamic free surface boundary condition, which states that the pressure on the free surface must be constant across the air-sea interface:

\begin{equation}
    -\frac{\partial \Psi}{\partial t} + \frac{1}{2}[(u)^2 + (w)^2] + g\eta = b(t) \quad \text{on } z = \eta(x, t) \quad (8)
\end{equation}
where $g$ is gravity and $b$ is a time dependent constant. The third boundary condition is the bottom boundary condition, which states that assuming that the bottom is impermeable and does not move with time, vertical fluid velocity at the bottom must be zero:

$$w = 0 \text{ on } z = -h \quad (9)$$

The final boundary condition is the periodic lateral boundary condition, which states that the wave must repeat after one wavelength and one period:

$$\Psi(x, t) = \Psi(x + \lambda, t) \quad (10)$$

$$\Psi(x, t) = \Psi(x, t + T) \quad (11)$$

The solution to this differential equation using the four boundary conditions yields the velocity potential equation:

$$\Psi(x, z, t) = \frac{Hg \cosh k(h + z)}{2\omega \cosh kh} \cosh kx \sin \omega t \quad (12)$$

where $\omega$ represents radian frequency and $k$ represents radian wavenumber, as well as the dispersion relationship:

$$\omega^2 = gk \tanh kh \quad (13)$$

which defines the relationship between $\omega$ and $k$, for any particular $h$. 
In deep water \((h > \lambda/2)\), within the water column, individual water parcels move in a circular path. These circular paths are called orbitals. Figure 4 shows a representation of these orbitals. As a wave propagates no mass is transported with it, however the wave transports energy. The amplitude of these orbitals diminish with depth, and at depths greater than approximately \(0.5\lambda\) no wave induced orbital motion is present. The instantaneous velocity underneath a wave propagating in the \(x\)-direction can be derived from equations 4, 6, and 12. The velocity in the horizontal direction can be written as:

\[
u = \frac{H}{2} \omega \cosh k(h + z) \cos(kx - \omega t)\tag{14}
\]

and the vertical velocity is:

\[
w = \frac{H}{2} \omega \sinh k(h + z) \sin(kx - \omega t)\tag{15}
\]

For \(0.05\lambda < h < 0.5\lambda\), the wave orbitals begin to elongate and form ellipses due to the interaction of the wave with the sea floor, these are referred to as intermediate water waves. When \(h < 0.05\lambda\), the wave becomes a shallow water wave, and the orbitals are flattened ellipsoids that reach the seafloor. In shallow water, the horizontal velocity component is much larger than the vertical component, and near the sea floor the wave velocities are oscillatory in the horizontal direction with little to no vertical component. As the wave enters very shallow water, the shoaling process occurs, where the wavelength and phase speed of the wave decreases while the wave height increases. Wave height will continue to
increase as depth decreases until the wave becomes unstable (i.e., reaches a critical steepness) and breaks.

2.1.2. Departures from Linear Wave Theory

Wave breaking and wave-bottom interactions are nonlinear phenomenon. Multiple waves propagating in the same space will interact (i.e., exchange energy) and change phase velocities nonlinearly due to the presence of the other waves (Longuet-Higgins and Phillips, 1962). Linear wave theory can be accurately assumed under conditions in which no breaking is occurring, wave steepness is small ($ka \ll 1$), and the interactions between multiple wave systems is limited.

However, wave interactions from multiple wave systems (e.g. swell and wind seas) are common, and often when in shallow coastal water, linear wave theory is not entirely valid due to wave interactions with the sea floor.

Group line associated energy makes up a non-trivial amount of energy in wavenumber-frequency ocean surface spectra. Plant and Farquharson (2012) found that the most likely source of this energy is wave interaction/interference induced breaking. Eliminating this energy from wave field reconstructions most likely has an adverse effect on the accuracy of phase-resolved wave field reconstructions.

2.2. Ocean Wave Measurements

This section outlines conventional sensors for measuring ocean surface waves as well as the use of radar as a remote sensor for measuring ocean waves. Details of the physics of radar measurements of the sea surface are provided. Conventional methods of processing
radar measurements of ocean surface waves using Fourier-based dispersion filtering are described. Finally, previous research on the application of POD to idealized radar measurements of synthetic ocean surface waves is described.

2.2.1. Conventional Wave Sensors

Ocean surface waves are conventionally measured using instrumentation such as wave buoys or acoustic velocity profilers. These devices record time series measurements at a single point in space, and use FFTs to calculate a frequency power density spectra.

Buoy displacements are either measured directly by used of GPS receivers in the case of GPS type buoys, or calculated from acceleration in the case of accelerometer buoys. Accelerometer buoys measure accelerations using accelerometers, resulting in measurement of angular accelerations in all six-degrees of freedom (roll, pitch, yaw, heave, surge, sway). The measured vertical acceleration (heave) is double integrated to calculate vertical displacements. From the time series of vertical displacements, a wave power spectrum is calculated. Wave direction is calculated using recorded pitch and yaw measurements in conjunction with an onboard compass. GPS based buoys measure (3D) velocity through the change of GPS measured position over time. Note that the change in GPS position is more accurate than absolute position. Typically wave buoys will record twenty minutes of data per hour, assuming that over the course of an hour changes in the wave field will be negligible (or in other words the wave field is stationary on time periods of one hour).

Acoustic wave velocity sensors such as acoustic Doppler current profilers (ADCPs), or acoustic wave and current meters (AWACs) transmit an acoustic signal that is reflected by
scatterers in the water (e.g., biological materials, sediments), which are assumed to move with the same velocity and direction as the water. A set of receivers measures the reflected acoustic wave and based on changes of its properties (e.g., phase/frequency) a Doppler shift is computed, which is related to the water (scatterer’s) velocity along that direction. By timing expected return signals from different distances (referred to as range-gating), vertical profiles of these velocities can be measured. The geometry of the receivers is designed such that by combining Doppler shifts measured by the different receivers, velocities in all three directions are obtained. Time averaging these measured velocities over time periods longer than the period of the surface waves can be interpreted as the ocean currents. The difference between this averaged velocity and the instantaneous velocity is predominantly the wave orbital velocities. Spectra of the wave velocities can then be computed from these time series for computation of wave statistics, and the wave orbital velocities and/or spectra can be converted to a wave height spectrum using linear wave theory (Dean and Dalrymple, 1991). Typically acoustic sensors are also set to record water velocities for 10-20 minutes per hour to conserve data space and battery power under the same assumptions as accelerometer-based buoys.

2.2.2. Radar Wave Measurements

Radar is a relatively new wave sensing technology. Over the last several decades this technology has developed into a more robust wave sensor. The next sections review these developments and state-of-the-art in this field.
2.2.2.1. Radar Scattering from the Sea Surface

Two primary categories of marine radar exist: coherent and non-coherent systems. Radar frequencies are typically in the X band (8 to 12 GHz). Radar measurements of the sea surface involve a radar system transmitting an EM wave that is reflected and scattered off of the sea surface back to a receiver typically collocated with the transmitter. Non-coherent systems measure backscatter intensity of the return signal only while coherent systems measure the phase of both the transmitted and received EM waves, and thus are able to calculate a Doppler shift based on the phase shift between the outgoing and incoming EM wave (Hwang et al., 2010). Coherent systems are preferable for ocean surface wave measurements due to a well-defined relationship between orbital velocities and wave height based on linear wave theory (Equations 14 and 15). In contrast, the relationship between backscatter and wave height is more complex and less well understood (Nieto Borge et al., 1999). Consequently, calculation of wave height from non-coherent systems requires the use of empirical transfer functions, which have to be developed for specific radar systems and locations (Nieto Borge et al., 2004). This consideration must be weighed against the fact that coherent systems are much more costly.

Marine (X-band) radar have an EM wavelength of approximately 3 cm. Therefore the ocean waves that produce a strong return signal are capillary waves, which are of similar wavelength, due to the Bragg resonance phenomena (Wright, 1966). When there is very little wind, and thus a glassy sea surface with little to no capillary waves, the radar return signal is insufficient for measurement of sea clutter (i.e., not enough power is returned to the receiver). Only under sufficient wind conditions, with enough capillary waves can radar measurements of the sea surface be made (Young et al., 1985). At low grazing angles, other
factors such as wave shadowing (shown in Figure 5) and trapping of EM waves behind a wave peak become significant as well (Valenzuela, 1978).

The primary signal oscillation in non-coherent radar systems is assumed to be due to tilt-modulation, which is associated with varying angles of incidence of the transmitted EM wave with respect to the free surface (Figure 5). Under this assumption, the peak return signal would occur near maximum wave slope, and minimum return signal would occur at peaks and troughs of the wave (Johnson et al., 2009).

2.2.2.2. Conventional Processing of Radar Backscatter

Radar backscatter from a non-coherent rotating radar system is conventionally processed by calculating a three dimensional PSD of the backscatter. The resulting spectrum is then filtered using the linear dispersion relationship (Equation 13) to eliminate non-wave contributions to the signal. The resulting filtered 3D spectrum can be integrated over $k_x$ or $k_y$ to produce a 2D spectrum that is a function of radian wavenumber ($k$) and radian frequency ($\omega$) (Young et al, 1985) and/or double-integrated to obtain a 1D spectrum in either wavenumber or frequency. An empirical transfer function is needed to convert any of these spectra (1D-3D) to a wave height spectrum (Hwang et al, 2010). From a wave height spectrum, wave statistics such as $H_s$, $T_p$, etc., can be calculated (Dean and Dalrymple, 1991; Earle, 1996). Application of an IFFT converts the spectrum back to the spatiotemporal domain.
2.2.2.3. Conventional Processing of Doppler Measurements

Two different methods can be used to calculate Doppler velocity from a coherent radar system: pulse pair and FFT-based methods. Pulse pair processing uses a Hilbert transform of the radar return and phase difference to calculate Doppler velocity. Doppler velocity can also be calculated using spectral processing of the complex radar return signal (Miller and Rochwarger, 1972; Thompson and Jenson, 1993). After the removal of the mean velocity, which is due to the ocean currents and phase-speed of the capillary waves, the primary oscillation of Doppler velocities is assumed to be due to the orbital velocities of the ocean waves. Doppler measurements of the ocean wave field are processed similar to radar backscatter. However, no empirical transfer function is needed, as the known relationship between orbital velocity and wave height can serve to convert from PSD of wave velocity to a wave height spectrum (equations 14 and 15), eliminating potential errors in an empirical transfer function. It should be noted that the measured Doppler velocity is the velocity along the direction of the measurement; thus, only in the up-wave and down-wave directions would the full orbital velocity magnitude be observed because in other orientations the projected component would not be the full magnitude of the orbital velocity.

2.3. POD Applied to Ocean Surface Wave Measurements

In this study, in contrast to the conventional techniques discussed in sections 2.2.2.2. and 2.2.2.3., POD, described in detail in section 4.3., is applied to radar ocean surface measurements to extract the ocean surface wave field from leading POD modes. Because this is a novel approach, few studies have examined POD applied to ocean surface
measurements. However, in a previous study (Hackett et al., 2014), POD is applied to idealized radar measurements of a simple sinusoidal wave model of a single frequency as well as linear superposition of multiple waves with different properties, and a Bretschneider spectral wave model. The results show that a single frequency wave model yields two dominant sinusoidal, complex conjugate modes as indicated by their much larger singular values. Together these two modes accurately reconstruct the single frequency wave model both with and without random noise, as shown in Figure 6 (adapted from Hackett et al., 2014).

The POD modes resulting from the application of the method to a single frequency wave are periodic and have wavelengths that are related to the wavelength of the measured wave. Two linearly superimposed waves result in 4 dominant POD modes. The leading mode pair reconstructs the wave with a higher amplitude or orbital velocity magnitude, and the second two dominant modes reconstruct the smaller of the two waves. The superimposed waves can be successfully separated on the basis of the POD modes exclusively as long as sufficient difference in frequency and wave height or orbital velocity magnitude is present.

POD was also applied to a wave field generated using a Bretschneider spectral wave model composed of 200 frequency components. The Bretschneider model contains frequency and directional bandwidth that distinguishes it from the simple single frequency wave models examined previously. Incorporation of these bandwidths increased the number of modes needed to accurately reconstruct the wave field. Figure 7 (reproduced from Hackett et al., 2014) shows the root mean square error (RMSE) normalized by significant wave height of POD reconstruction of the Bretschneider wave model using various numbers of modes versus the difference between radar look direction and wave propagation direction. In order
to obtain errors less than 10%, 25 modes are needed and reconstructions of 100 modes produce errors of less than 1%. Recall that the wave field was generated using 200 spectral components; thus, the POD method was able to replicate the same wave field with less than half the amount of basis functions as a Fourier transform.

The results of the application of POD to idealized radar measurements of a simple sinusoidal wave model and Bretschneider spectral model show that POD is a viable technique to extract ocean surface wave fields from radar measurements because the mode basis functions can be linked to the physical properties of the measured wave field. The advantages of POD over FFTs include less computational demand, and the POD technique, while based on linearity, is optimal over other linear techniques for representation of nonlinear phenomena (Hackett et al., 2014). The application of POD to radar measurements of ocean waves, and direct comparison to FFT techniques has not been previously examined and is the focus of this study.
Figure 1: Schematic defining various wave parameters. Adapted from Dean and Dalrymple, 1991.
Figure 2: Qualitative wave power spectrum, showing range of wave periods and their forcing mechanisms. Reproduced from Kinsman, 1965.
Figure 3: Schematic of wind waves versus swell waves. Adapted from Dean and Dalrymple, 1991.
Figure 4: Black circles represent orbitals. In deep water, orbitals decrease in size to a depth of approximately $0.5\lambda$. (http://www.indiana.edu/~g105lab/1425chap12.htm).
Figure 5: Schematic of tilt modulation and shadowing. For this figure only, $R$ is range, $\Lambda$ is altitude of the radar above sea level, $\Theta$ is the grazing angle of the radar, $u$ is reflected energy, and $n$ is the normal to the wave surface. Reproduced here from Nieto Borge et al., 2004.
Figure 6: The black curves show the root mean squared error (RMSE) normalized by wave amplitude ($A_0$) for the two-mode reconstruction with respect to the difference between look angle and propagation direction for various noise levels (see legend). The red lines show the accuracy of the noise reconstructed using the remaining modes. Reproduced here from Hackett et al., 2014.
Figure 7: RMSE normalized by significant wave height of the POD reconstruction of a Bretschneider wave model using various numbers of modes and varying look direction relative to the direction of wave propagation. Reproduced from Hackett et al., 2014.
3.0. Experimental and Numerical Radar Data

To evaluate the POD method in comparison to FFT-based methods as well as to conventional wave measurements (e.g., wave buoys), radar data from three different experiments and one numerical model are used. Multiple data sources from different radar systems over a variety of environmental conditions allow a comprehensive evaluation of the method. Numerical data are used to complement these experimental data because the numerical data has a known solution for the wave field at every point in space and time for comparisons between the methods, which is a luxury not afforded in the experimental data. Conversely, the numerical data are simulated and therefore inherently involve approximations, empirical relationships, and other simplifications. Thus, a combination of both numerical and experimental data is used to evaluate the method. A description of the experiments, radar systems used for data collection, and sub-selection of particular datasets from these sources is provided in section 3.1. Section 3.2. describes the initiation of the numerical model and comparisons with measured data.

3.1. Experimental Radar Data Sources

First, experimental datasets are described followed by dataset sub-selection.
3.1.1. Scripps Institute of Oceanography Pier Experiments

The DREAM radar system, manufactured by Sensor Concepts Inc., was installed at the end of the Scripps Institute of Oceanography (SIO) pier in La Jolla, CA, where data were collected from the 26th through the 30th of July 2010. The DREAM radar is a calibrated, linear, coherent, dual-polarization, X and Ku band Doppler system. The radar system measured Doppler velocity along a 1D range transect over time. The transect was determined by visually pointing the radar through a radar boresight into the direction of wave propagation, which was predominantly perpendicular to shore. Doppler velocity was calculated from the phase change between the transmitted and received signal using the pulse-pair method (Hwang et al., 2010; Miller and Rochwarger, 1972). These 1D range transects, collected in time, result in a 2D spatiotemporal distribution of Doppler velocity. An example of these data is shown in Figure 8.

Radar data was collected at both X and Ku-band for both horizontal-transmit and horizontal-receive (HH) polarization as well as for vertical-transmit and vertical-receive (VV) polarization. This experiment resulted in collection of 13 datasets for each frequency and polarization. However, for this study, only the VV and X-band data are used because it is known that VV polarization is better for capturing the wave field (e.g., Hackett et al., 2015). The X-band data was chosen because the data from the other experiments (see Sections 3.1.2-3.1.3) was collected in X-band; thus, using the X-band data from this experiment makes the comparison between the experiments more straightforward. The spatial footprint covered by the radar was 614.5 m at a spatial resolution of 30 cm. The data were collected for 10 minutes at a sample interval of 0.0013 s (pulse repetition frequency (PRF) of 800 Hz). The Doppler data were subsequently low pass filtered
(described in Hackett et al., 2011) to reduce high frequency noise and then down sampled to a resolution of 0.25 s. This down-sampling will not adversely affect wave measurements as the Nyquist frequency of the down-sampled data is 2 Hz, and the highest wave frequency of interest is approximately 0.25 Hz. This down-sampling does not introduce aliasing because the data were low pass filtered, removing high frequency content, before the down-sampling. Table 1 provides the radar configuration for these datasets. Mini GPS wave buoys, developed by the Coastal Observing Research and Development Center at SIO, were used to collect ground truth data. More information about the DREAM system and data collection for this experiment may be found in Hackett et al. (2011) and Hackett et al. (2012).

3.1.2. R/V Melville Experiment

The R/V Melville experiment was conducted aboard the R/V Melville from the 14th through the 17th of September 2013, south of the Channel Islands offshore of Los Angeles, CA. Two radar systems were used during this experiment; the first was developed by the University of Michigan (UM) and the Ohio State University (OSU), which will be hereafter referred to as the UM radar (Alford et al., 2015), and the second by the company Applied Physical Sciences (APS), which will be hereafter referred to as the APS radar (Connell et al., 2015). During the experiment 12 GPS mini-buoys, developed by the Coastal Observing Research and Development Center at SIO, were deployed for use as ground-truth wave sensors (Drazen et al., 2016). A brief description of the radar systems used in this experiment are provided below.
3.1.2.1. UM System

This coherent-on-receive (Smith et al., 2013) radar has a center frequency of 9.41 GHz, VV polarization, and rotates at 24 RPM. Pulse-pair processing is used to estimate Doppler velocity (Miller and Rochwarger, 1972). Data is a function of range \( r \), time \( t \), and azimuth \( \phi \). The Doppler estimates are averaged over 12 pairs for noise reduction, yielding a Doppler velocity range distribution approximately every 0.86° of rotation. The resulting Doppler velocity distributions cover a range of 960 m at a resolution of 3.75 m. One full revolution of the radar system produces a one radar frame. An example Doppler velocity frame is shown in Figure 9. Each dataset is approximately 3 minutes in length and hours of data were collected under a variety of environmental conditions. A summary of the radar parameters for this system are shown in Table 1. The radar had a blanking range of 100 m around the vessel to eliminate high power return.

3.1.2.2. APS System

The APS system uses four coherent antennas mounted at 90° to each other rotating at 5 RPM. Each of the four antennas are identical. Each has a center frequency of 9.2 GHz and PRF of 25 kHz. Data is a function of range \( r \), time \( t \), azimuth \( \phi \), and antenna \( A \), \( D(r,t,\phi,A) \). FFT processing (Thompson and Jenson, 1993) is used to produce Doppler estimates over 64 pulses, yielding a Doppler range distribution every 1.23° for a rotation rate of 12 seconds. One quarter rotation of the system yields a complete frame of data every 3 seconds because the data from each of the four antennas is combined to generate one frame. The potential advantage of this configuration is that a slower rotation rate permits more pulses to go into each Doppler estimate (referred to as the dwell time), which should
make the Doppler estimate more accurate. The tradeoff is the slow rotation rate can result in aliasing because the re-visit time to the same patch of ocean surface is longer than many of the ocean surface wave periods. The four antennas mitigate this tradeoff. The resulting Doppler distributions have a range resolution of 4.8 m and cover a range of 998 m. An example frame, generated by patching data from all four antennas together, is shown in Figure 10. Each dataset is about 2 minutes in length and data were collected in the same variety of environmental conditions as the UM data from this experiment. The parameters for this radar system are summarized in Table 1. The radar had a blanking range of 100 m around the vessel to eliminate high power return.

3.1.3. Culebra Koa 2015 Experiment (CK15)

The CK15 experiment test took place from the 17th-21st of May 2015, off the east coast of Oahu, Hawaii. This was a joint military seabasing exercise, featuring the USNS Montford Point, a new class of ship called a Mobile Landing Platform (MLP), made to load and unload cargo from other ships while at sea. US Navy, Marine, Air Force, and Army personnel all took part in the exercise. During the experiment radar data were collected using the same UM and APS radar systems detailed in the above sections. The radars were mounted on the USNS Dahl, a Watson class Large, Medium-Speed, Roll-on/Roll-off (LMSR) vessel. Hours of data were collected in a variety of environmental conditions. During the experiment, 6 GPS mini-buoys were deployed and used as ground-truth wave sensors (Drazen et al., 2016).
3.1.4. Datasets Used in this Study

Data from these three experiments were sub-selected to evaluate the method over a range of environmental conditions for multiple radar system configurations. Two X-band VV polarization datasets from the SIO pier test were sub-selected for the study to examine the effect of sea state modality on the POD method. Only the two datasets used will be described in this section, however, detailed descriptions of the entire data collection are contained in Hackett et al., 2012. Wind speeds during these two runs were 3.9 m/s, which are high enough to generate sufficient surface roughness to enable wave measurements. Dataset 246 consists of a bi-modal wave field with both swell and wind-waves of approximately equal energy. Dataset 269 is strictly wind-wave dominated. Relevant wave field and environmental statistics during runs 246 and 269 were calculated from mini GPS wave-buoy data, and permanent sensors on the SIO pier, and are provided in Table 2.

In order to evaluate wave field retrieval methods over various environmental conditions for rotating radar systems, datasets from both the R/V Melville and CK15 experiments were sub-selected to cover a range of sea states, wind speeds, and number of wave systems. Sequential datasets from both the APS and UM radar were selected when available in order to evaluate the impact of radar system design on the wave field retrieval method. Datasets were selected to cover a range of small (less than 0.5 m), medium (between 1 m and 2 m), and large (greater than 2 m) $H_s$ for swell dominant, wind wave dominant, and mixed sea states. $H_s$ conditions as measured by mini GPS wave buoys for these experiments ranged from 0.1-2.2 m. Representative datasets of the highest and lowest $H_s$ are included in the sub-selected datasets. Swell dominant is defined as having both wind-sea and swell spectral peaks, with the larger peak in the swell period band defined as 7-20 s, while wind wave
dominant would have a larger peak in the wind wave period band (2-6 s). Because $U_w$ has a large effect on the SNR of the radar measurement (Rozenberg et al., 1999; Holman and Haller, 2013), the UM and APS datasets from both the R/V Melville and CK15 tests with the smallest and largest $U_w$ are also included in the sub-selection. Table 3 shows the environmental conditions as measured by GPS wave buoys ($H_s$, $\lambda_p$, and $V_{rms}$), ship-based sensors ($U_w$), and calculated from radar data ($\Delta \Theta$) as well as dataset number, associated test, date and time of the measurements, number of wave systems, and which radar systems had collected data.

### 3.2. Numerical Data

A numerical radar emulator model, developed by Gordon Farquharson who is a Principal Engineer at the *Applied Physics Laboratory* at the *University of Washington*, simulates various aspects of the radar scattering and geometry including range-dependent signal to noise ratio (SNR) and Doppler “sea spikes.” The latter are incorporated by evaluating wave slope criterion and applied when the wave slope exceeds a threshold for breaking; then Doppler velocities are instantaneously set to the phase speed of the wave rather than the orbital wave velocities. This emulator simulates a wave field and radar system, and then simulates the scattering from the ocean surface that would be produced from the interaction of that radar and wave field. Each emulator dataset is ten minutes in duration. The wave field data along with the simulated radar measurements allows reconstructed wave fields to be compared to a ground-truth at every place in space and time. An example of the emulator data is shown in Figure 11 (panels B and C).
Two numerical simulation runs are used for this study. The first run (emulator run 1) was initiated using the directional wave spectrum shown in Figure 12. This spectrum is calculated from GPS mini buoy velocity data measured during the R/V Melville experiment on September 17, 2013, starting at 15:45:26. The directional wave spectrum measured by the buoy is converted from frequency and direction to a $k_x$-$k_y$ directional spectra using linear wave theory (Tucker and Pitt, 2001; Young and Babanin, 2009). The environmental conditions during this time are similar to those in dataset 13 (Table 3), which was recorded approximately an hour before the buoy data used to initiate this numerical simulation. The UM radar parameters (Table 1) were used for the emulated radar configuration and processed to obtain Doppler velocity frames over time as described in Section 3.1.2.1.

The second emulator run (emulator run 2) was initiated using a directional spectrum which matched the previously discussed directional distribution (emulator run 1), however had a $H_s$ approximately 60% smaller than emulator run 1, and thus contains lower magnitude orbital velocities (Figure 11, panel B). Due to lower wave heights for the same frequency distribution, wave steepness is lower and little to no wave breaking is triggered in the numerical simulation. This reduction causes the $k$-$\omega$ spectrum of emulator run 2 to contain no group line (Figure 13, panel B).

Figure 11 shows a comparison of an example frame of UM Doppler velocity data from dataset 13 and an example frame of both numerically simulated Doppler velocity datasets. The non-group line emulator run 2 (Figure 11, panel B) shows overall lower Doppler velocities than the group line run 1 and UM dataset 13. The group line emulator run 1 shows Doppler velocity data of more similar magnitude to the UM dataset, but with more
Doppler sea spikes (due to wave breaking) than the UM data, as well as a non-uniform distribution of these spikes, with most located in the up-wave direction (positive x). It also shows more group line energy than that in dataset 13 relative to the energy on the dispersion relationship. The UM dataset and group line emulator run 1 spectra both show a non-negligible group line feature compared with dispersion curve energy, while the non-group line emulator run 2 shows no group line energy. The group line emulator run 1 spectrum also shows slightly higher overall energy. As shown in Figure 13, each spectra has a peak located on the dispersion curve at approximately 80 m wavelength, consistent with the dominant peak in the buoy directional spectrum. Presumably, this increase in the orbital velocities resulted in greater interference between the wave systems inducing more instances of breaking for the reasons described in Plant and Farquharson (2012), which creates more sea spikes in the simulated Doppler data, increased spectral energy, and strengthening of the group line energy. These results collectively demonstrate that while the numerical simulations were initiated with a similar wave field to that measured in the UM dataset, there are clear differences in the Doppler velocity measurements and k-ω spectra which should be considered when making comparisons between the numerical simulation and experimental results.
Table 1: Radar parameters: center frequency \( f_c \), bandwidth \( \Delta f_b \), polarization, resolution, footprint, pulse repetition frequency (PRF), and rotation rate (RPM).

<table>
<thead>
<tr>
<th>Radar System</th>
<th>( f_c ) (GHz)</th>
<th>( \Delta f_b ) (MHz)</th>
<th>Polarization</th>
<th>Resolution (m)</th>
<th>Radar Footprint (m)</th>
<th>PRF (Hz)</th>
<th>Rotation Rate (RPM)</th>
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<td>VV</td>
<td>0.30</td>
<td>615</td>
<td>800</td>
<td>0</td>
</tr>
<tr>
<td>UM</td>
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<td>VV</td>
<td>3.75</td>
<td>960</td>
<td>2000</td>
<td>24</td>
</tr>
<tr>
<td>APS*</td>
<td>9.20</td>
<td>28</td>
<td>VV</td>
<td>4.80</td>
<td>998</td>
<td>25000</td>
<td>5</td>
</tr>
</tbody>
</table>

*one of the four APS antennas
Figure 8: Example spatiotemporal Doppler velocity distribution from the SIO pier experiment collected using the DREAM radar.
Figure 9: Example frame of Doppler velocity data collected with the UM radar during the R/V Melville experiment. Each dataset consists of a time series of these images.
Figure 10: Example frame of Doppler velocity data collected with the APS radar during the R/V Melville experiment. Each dataset consists of a time series of these images.
Figure 11: Panel A shows an example frame from UM radar dataset 13, taken closest in time to the buoy measurements used to initiate the numerical simulation. Panel B shows an example frame from the non-group line emulator run 2. Panel C shows an example frame from the numerical simulation containing a group line, emulator run 1 (note the increased orbital velocities).
Figure 12: Directional wave spectrum (in $k_x$-$k_y$) used to initiate emulator run, which is based on data measured by a GPS mini buoy on September 17, 2012, at 15:45:26 during the R/V Melville experiment and has statistical properties similar to dataset 13 in Table 3.
Figure 13: Panel A, B, and C show the $k$-$\omega$ spectrum for the datasets shown in Figure 11, respectively. All spectra are calculated from a 3D FFT, integrated over $k_y$. 
Table 2: Wave field and environmental conditions for the two SIO Pier experimental datasets used in this study as measured by the GPS mini buoy and sensors on SIO pier. $H_s$ is significant wave height, $T$ is mean wave period, $T_p$ is peak wave period, $\Theta_p$ is peak wave direction, $\Theta_w$ is wind direction, and $U_w$ is wind speed.

<table>
<thead>
<tr>
<th>Run</th>
<th>Date</th>
<th>Radar Time</th>
<th>$H_s$ (m)</th>
<th>$T$ (s)</th>
<th>$\Theta_p$ (°)</th>
<th>$T_p$ (s)</th>
<th>$\Theta_w$ (°)</th>
<th>$U_w$ (m/s)</th>
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Table 3: Datasets sub-selected for this study from the R/V Melville and CK15 experiments and the associated range of environmental conditions covered. $H_s$, $\lambda_p$, and $V_{rms}$ were measured using GPS mini wave buoys. $U_w$ was measured using a ship-mounted anemometer. $\Delta \Theta$ was calculated from the radar measured directional wave spectrum. The dataset number, associated test, date and time of the measurements, number of wave systems, and which radar systems had collected data at this time are also provided.

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<th>$U_w$ (m/s)</th>
<th>$\lambda_p$ (m)</th>
<th>$V_{rms}$ (m/s)</th>
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<td>105</td>
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* Lowest $H_s$ for respective test
** Highest $H_s$ for respective test
++ Highest $U_w$ for respective test
++ Highest $U_w$ for respective test

44
4.0. Methods

Wave field extraction from radar data is typically performed using FFT-based dispersion filtering methods, like that introduced in the seminal paper of Young et al. (1985). POD wave field extractions (Hackett et al., 2014) are compared to this conventional Fourier filtering approach. The implementations of these methods are described in the following subsections. Preparation of the rotating radar datasets (described in section 3) are performed before wave field extraction methods are applied in order to geo-reference the data, re-orient the radar frame along the dominant wave direction (as opposed to the ship forward direction), detrend the data, and to convert the polar grid into a Cartesian coordinate system. These steps are primarily needed for the POD based wave field estimates and comparisons to ground truth sensors, but are also applied prior to the FFT based extraction to ensure all datasets are analyzed identically and that the results are comparable. The last subsections discuss the methods used for comparing the results of the FFT and POD methods to each other as well as to ground truth wave measurements (i.e., wave buoy data and emulator simulated wave fields).

4.1. Rotating Radar Data Preparation

Before either the FFT or POD wave field extraction methods are applied, the datasets are geo-referenced, detrended, converted to a Cartesian grid, rotated such that the dominant wave propagation direction is aligned with the $x$-axis, and have the area around the ship
blanked. The geo-referencing is performed because it is required for phase-resolved time series comparisons with buoy records. Determining where the buoy is located relative to the radar requires that both datasets be referenced to an Earth coordinate system. The polar Doppler velocity data \( D(r,t,\phi) \) is transformed to a Careesian grid \( D(x,y,t) \) before applying the FFT and POD methods. Sea clutter measurements are known to be sensitive to the look direction of the radar (Walker, 2001). It was found that the most accurate results of the POD method were obtained when the waves were traveling predominantly in the \( x \)-direction. The dominant wave direction is determined from the directional Doppler velocity wave spectrum as well as the time series of radar images due to the 180° directional ambiguity innate to the wavenumber spectrum of an individual radar frame (Young et al., 1985). Subsequently, the data are rotated such that the dominant wave propagation direction is aligned with the \( x \)-direction of the Cartesian grid. A box around the origin of the size of the radar’s minimum range is blanked with zeros in all directions.

The rotational radar Doppler measurements are a function of azimuth, range, and time: \( D(r,t,\phi) \), where the zero azimuth is oriented in the direction of the ship’s heading. First, the data are re-oriented northward by subtracting the ship’s heading, such that the zero azimuth is oriented northward. The Doppler data are then linearly detrended from the end of the blanking range to the maximum range used in this study (700 m) for each azimuth. A 700 m range limit is selected in this study to ensure analysis is limited to high signal-to-noise ratio (SNR) Doppler data.

The directional wave spectrum is calculated by a 3D FFT applied to the time series of radar frames. The resulting spectrum is integrated over \( \omega \) to yield a 2D directional wave spectra as a function of \( k_x \) and \( k_y \). An example 2D directional spectra from UM dataset 10 is shown.
Because a 180° directional ambiguity exists in a single frame radar calculated wavenumber spectrum (Young et al., 1985), the time series of radar images is also used to confirm the direction of propagation. The peak wave direction is determined as the angle with the largest wave energy and the data \( D(r,t,\phi) \) is rotated such that \( \phi=0 \) is oriented in the peak wave direction. Each radar frame (one complete rotation) is transformed to a Cartesian grid with 10 m spacing using nearest neighbor interpolation, such that the center of the grid \((x,y) = (0,0)\) is the location of the radar. The process results in a 3D stack of radar frames in a Cartesian coordinate system, i.e., \( D(x,y,t) \) (an example stack of radar frames is shown in Figure 15). Fourier dispersion curve filtering (section 4.2) and POD wave field extraction methods (section 4.3) are subsequently applied.

### 4.2. Conventional FFT Technique

Conventional FFT-based wave field extraction techniques are applied to the rotating radar data for comparison to the POD method. The conventional processing technique outlined here is similar to the method of Young et al. (1985). A 3D matrix comprised of sequential rotating radar frames in space and time is compiled, \( D(x,y,t) \) as described in section 4.1. The number of available frames, \( N \), is dependent on the dataset, but typically \( N=40-60 \) frames, 2-3 minutes in time containing multiple wave cycles. The data consist of 141 range bins in \( x \) and \( y \). The first 32 radar frames are used to construct a 141x141x32 stack of radar frames (Figure 15 shows an example stack of radar frames), which is zero-padded to a size of 256x256x32. A 3D FFT is applied to the zero padded dataset. The Fourier coefficients are subsequently filtered using a 3D binary dispersion relationship filter \( d_k(k_x,k_y,\omega) \):
\[ d_k = 1 \quad \text{for } \omega - W \Delta \omega < \sigma < \omega + W \Delta \omega \]  
\[ d_k = 0 \quad \text{otherwise} \]  

where \( \omega \) is radian frequency, \( \Delta \omega \) is the radian frequency resolution, \( W \) is filter width, and \( \sigma \) is the linear deep water dispersion relationship including current \((U)\):

\[ \sigma = U \cdot k + \sqrt{g k} \]

where \( U \) is the current, \( k \) is radian wavenumber, and \( g \) is gravity. Equation (16) for a filter width of \( W = 1 \) is shown in Figure 16. This filter eliminates energy that does not adhere to the linear dispersion relationship for ocean surface waves and acts to remove the non-wave field contributors to the radar measurements (i.e., it isolates the linear wave signal). An inverse Fourier transform (IFFT) is then applied to the filtered Fourier coefficients to return the data to \( D(x,y,t) \). The center frame of the stack is extracted and saved as the phase-resolved wave orbital velocity map for the center time of the stack. Subsequently, another zero-padded stack of data is created by shifting the set of 32 frames forward by one frame in time and the process repeats until the last frame in the stack is frame \( N \). The result is a time series of phase-resolved wave orbital velocities maps that spans frames 16 to \( N-16 \).

4.3 Proper Orthogonal Decomposition (POD)

POD is a method used frequently in analysis of complex, nonlinear signals, such as turbulence. The method enables low dimensional representations of complex high dimensional signals (Chatterjee, 2000). Reconstruction using a sub-selection of modes can
act as a filter for the signal of interest. The POD method described in this section is adapted from Hackett et al. (2014).

The POD method takes a signal, in the case of the rotating radar data, one Doppler velocity radar frame, \( D(x,y) \) in which \( x \) and \( y \) are spatial coordinates; or, in the case of the non-rotating radar data, \( D(r,t) \) in which \( r \) is a spatial coordinate and \( t \) is a temporal coordinate, and decomposes the signal into a series of orthonormal basis functions and temporal (or spatial) coefficients. The basis functions are referred to as modes. The shape of the mode is determined by the data itself and is not an assumed function \( a \) priori. The modes are ranked such that the first mode accounts for the most variance of the signal, the second mode – the second order contributor to the variance, and so-on. A summation of all the modes multiplied by the corresponding coefficients results in the reconstruction of the original signal. A singular value decomposition is used to perform the POD:

\[
D = B\Sigma P^T
\]

where \( B, \Sigma, \) and \( P \) are matrices, and the superscript \( T \) indicates matrix transpose. The mode functions are encompassed in \( P \), and the diagonal elements of \( \Sigma \) are the singular values of matrix \( D \). Let \( Q = B\Sigma \), then,

\[
D = QP^T = \sum_{k=1}^{M} q_k p_k^T
\]

where \( q_k \) are the spatial or temporal coefficients of the signal, and \( p_k^T \) are the basis functions of the Doppler velocity, or the proper orthogonal modes. The singular values occur in
ranked order along the diagonal elements of \( \Sigma \) and signify the relative importance of each mode.

A low-order representation of the signal is obtained by reconstructing the Doppler velocity with a subset of the mode functions and time (or spatial) series coefficients. By associating particular modes with particular physical characteristics of the waves, this technique could be used to filter, or extract, the wave field signal from the radar measurements, which also contains contributions from other sources aside from the wave field. Some of these artifacts include: shadowing, sea-spikes, and range decay (Smith, 1996; Walker, 2001; Nieto Borge et al., 2004; Raynal and Doerry, 2010; Plant and Farquharson, 2012). Evaluation of whether or not the mode functions can be used as a filter in this way is a primary component of this research.

The reconstructed Doppler velocities, based on this sub-selection of modes, are considered phase-resolved wave orbital velocity maps. For non-rotating radar data, the reconstruction results in a single range-time distribution of Doppler velocity. For the rotating radar data, this procedure is applied to all radar frames in the time series; thus, the result is a time series of phase-resolved wave orbital velocity maps covering frames 1 to \( N \). For purposes of comparison to the FFT-based approach only the results from frame 16 to \( N-16 \) are considered (see section 4.2).

4.4. Statistical Evaluation

In order to evaluate and compare the accuracy of wave field extraction methods, wave statistics are computed from GPS mini-buoys and from radar extracted wave orbital
velocities. Statistics compared are $H_s$, $V_{rms}$, and $\lambda_p$ (significant wave height, root-mean-squared (RMS) orbital velocity, and peak wavelength, respectively).

Buoy statistics are calculated from the time series of water velocities from each available buoy during the radar data collection times. Statistics computed for each buoy are averaged over all available buoys in order to account for inhomogeneity of the wave field over the radar footprint. Sea surface displacement is calculated from the integral of the high pass filtered (frequency cutoff of 0.05 Hz) $w$ velocity component of each buoy time series. $V_{rms}$ is the RMS of this filtered $w$ time series.

In order to calculate $H_s$, a 1D spectrum is computed from the $w$ time series. The 1D spectrum is converted from orbital velocity spectra to a sea surface elevation spectra (Hwang et al., 2010). Before the conversion of velocity to sea surface elevation spectra (for radar spectra), the velocity spectrum is converted from a wavenumber to frequency spectra following the method of Plant (2009). The spectra are converted using:

$$\frac{S(k)}{c_g} = S(\omega)$$

where $c_g$ is the group velocity; which is computed as $c_g = d\omega/dk$ using the still water dispersion relationship (Equation 13; section 2.1.1.). Based on linear wave theory, the relationship between velocity and sea surface elevation spectra is:

$$S_v(\omega) = \int_0^{2\pi} \frac{g^2 k^2 \cosh^2[k(h+z)]}{\omega^2 \cosh^2(kh)} S_\eta(\omega) D_\theta(\theta) \cos^2(\theta - \theta_r)d\theta$$

(21)
where \( D_\theta \) is a direction distribution, \( \theta \) is the direction of wave propagation, \( \theta_r \) is the radar look direction and \( z \) is the vertical coordinate, which is defined as zero at the mean free surface. Subscript \( v \) indicates a velocity spectrum, and subscript \( \eta \) indicates a sea surface elevation spectrum. Assuming measurements were performed at the sea surface \( (z = 0) \), the radar was looking into the wave propagation direction \( \theta - \theta_r = 0 \), and neglecting directional effects, Equation 21 simplifies to the following equation:

\[
S_\eta(\omega) = S_v(\omega) \frac{\tanh^2(kh)}{\omega^2}
\]  

(22)

\( H_s \) is calculated as (Earle, 1996):

\[
H_s = 4\sqrt{m_0}
\]  

(23)

where:

\[
m_0 = \int S_\eta(f) df
\]  

(24)

where \( f \) is linear frequency. Equations 23 and 24 are appropriate for deep water ocean surface waves, and the water depth for the experiments discussed in section 3 are either close to or well within the deep water limit. Upper and lower 90% confidence intervals for significant wave height are estimated using the method outlined in the NDBC manual (Bendat and Piersol, 1980; Donelan and Pierson, 1983; Earle, 1996). The confidence interval is dependent on the total degrees of freedom \( (T_{DF}) \), which is dependent on the width of the spectrum:
\[
T_{DF} = \frac{2\left(\sum_{i=1}^{M} S_i(n_i)^2\right)}{\sum_{i=1}^{M} (S_i(n_i))^2}
\]  

The upper and lower confidence intervals \((H_{ci})\) are:

\[
H_{ci} = H_s \left( \frac{T_{DF}}{\chi^2(T_{DF}, 1 \pm \alpha/2)} \right)^{1/2}
\]

where \(\chi^2\) values are obtained from chi-squared probability distribution tables and \(\alpha = 0.1\).

For 90% confidence intervals, \(\chi^2\) is calculated using (Earle, 1996):

\[
\chi^2 = T_{DF} \left\{ 1 - \frac{2}{9T_{DF}} \pm 1.645 \left( \frac{2}{9T_{DF}} \right)^{1/2} \right\}^3
\]

The peak wavelength \((\lambda_p)\) is found by locating the frequency in the 1D spectrum with the highest energy and converting to wavelength using the method outlined in Plant (2009).

Wave statistics for rotating radar-extracted wave fields are computed from the time series of phase-resolved wave orbital velocities for the datasets in Table 3 (section 3). Statistics are calculated along 1D range transects in the peak wave direction ±5 degrees. Statistics are calculated independently for each transect and then averaged over all transects that span ±5 degrees around the peak wave direction. These statistics are computed along the peak wave direction because Doppler radar measures a projection of the total velocity along the radar look direction; thus, the measured Doppler velocity only contains all of the orbital velocity along the wave propagation direction. When two wave systems are present, two
different bearings could contain the maximum orbital velocity for the wind seas and swell (assuming they are not propagating in the same direction). Thus, we chose to compute the statistics along the radar identified peak wave direction with the understanding that any secondary wave system will only be encompassed as a projection of their orbital velocity along that (primary system) direction. This procedure is also adopted because in the conversion between orbital velocity and wave height (Equation 22) it is assumed that the radar is looking into the wave propagation direction. $V_{rms}$ is calculated from the RMS orbital velocity along each transect. $H_s$ and $H_{ci}$ are calculated using the same method described above for the buoy data based on the 1D spectrum along each 1D transect of radar-based orbital velocities. $\lambda_p$ is also determined in the same way as the buoy data but from the 1D spectrum of orbital velocities over range averaged over all transects (i.e., $\phi$ of $\lambda_p \pm 5^\circ$).

4.5. Phase-Resolved Analysis

Phase-resolved radar extracted wave fields are compared to buoy time series for datasets in which buoy(s) are within the field of view of the radar. Zero lag cross correlation ($C$) and $E_{rms}$ (root mean squared error) are the metrics used for comparisons:

$$C = \frac{\sum_{i=1}^{n}(V_{bi} - \bar{V}_b)(V_{ei} - \bar{V}_e)}{(n-1)sBs_e}$$  \hspace{1cm} (28)

$$E_{rms} = \sqrt{\frac{\sum_{i=1}^{n}(V_{bi} - V_{ei})^2}{M}}$$  \hspace{1cm} (29)

where $M$ is the number of data points in the time series being compared, $V_b$ is the GPS mini wave buoy measured orbital velocity, $\bar{V}_b$ is the mean of the GPS mini wave buoy velocity
time series, $V_e$ is the estimated orbital velocity from either the POD or dispersion filtering method, $\bar{V}_e$ is the mean of the POD or FFT-based estimated orbital velocity time series, $s_b$ is the standard deviation of the buoy orbital velocity time series, and $s_e$ is the standard deviation of the POD or FFT-based estimated orbital velocity time series. Comparing phase-resolved time series rather than statistical comparisons enables evaluation of how well the phasing of the waves is resolved with the two wave field extraction methods.

At the time of each radar frame, latitude and longitude from GPS mini-buoys is used to locate the nearest data-point in the geo-referenced radar wave field. Due to GPS accuracy limits, the nearest radar range bin may not encompass the buoy but it should be within this radar range bin or one of the adjacent ones. Consequently, the velocity in the closest range bin as well as the immediately adjacent range bins are searched for the closest match to the buoy velocity, and the best match is selected to construct a radar-based orbital velocity time series. Although the GPS mini buoys have a temporal resolution of 1 Hz, the data are down-sampled to match the temporal resolution of the radar time series to enable an equivalent comparison. Zero lag cross correlation coefficients ($C$; Equation 28) and RMS error ($E_{rms}$; Equation 29) are calculated between each available buoy velocity time series and the POD and FFT based velocity time series, and then averaged over all available buoys.
Figure 14: Example radar directional wave spectrum for UM radar dataset 14. Note the 180° directional ambiguity innate to a single radar frame measured wavenumber directional spectrum (Young et al., 1985)
Figure 15: Example stack of radar frames from UM radar dataset 14.
Figure 16: 3D linear wave dispersion relationship.
5.0. Results

This section discusses results of the three main goals of the study: (i) the shape of the POD mode functions in relation to the physics of the measured wave field, (ii) comparison of wave field statistics between POD reconstructions and conventional dispersion filtering methods, and (iii) to compare accuracy of phase-resolved wave fields computed from both methods as well as a discussion of the results in the context of other studies.

5.1. Physical Significance of POD Modes

POD mode basis functions are derived from the data \textit{a posteriori}, because of this they are not restricted to sines and cosines as in Fourier methods. The interpretation of POD mode functions however, is more complex than Fourier methods, as there is no inherent physical interpretation of the mode functions. This section will examine the physical interpretation of the POD basis functions as applied to Doppler radar measurements of the ocean surface. Mode basis functions from staring or non-rotating radar, rotating radar, and emulated rotating radar are examined in this section.

5.1.1. Staring Radar

The physical interpretation of POD mode functions as related to wave field physics from the staring radar SIO experiment are examined. This experiment is chosen to examine first because the data format (i.e., $D(r,t)$) most closely matches the idealized radar measurements of simulated 2D wave fields examined in Hackett et al. (2014), which
showed that leading POD modes shared characteristics of the wave field physics. To investigate the effect of the number of wave systems present, two datasets were selected to examine. Dataset 246 was collected under mixed seas with both swell and wind-wave energy present, while dataset 269 was collected during wind sea only conditions.

Figure 17 shows the power spectral densities (or energy spectrum) of each of the leading twenty mode functions from POD applied to spatiotemporal Doppler velocity distributions in range and time (see Figure 8; section 3) for dataset 269 during wind sea only conditions. The vertical dashed line shows the peak wavenumber ($\lambda_p$) measured by the GPS mini wave buoy during the time period that the data was collected. All of the leading modes, with the exception of mode one, show spectral peaks at or surrounding the buoy measured peak wavelength, which indicates that a significant portion of the variance of the mode functions are associated with wavelengths near the measured wave field’s peak wavelength. Figure 18 shows the second mode function, further confirming that the function is oscillatory with a wavelength that is similar to the peak wavelength of the measured wave field. It can be deduced that under uni-modal sea conditions the leading POD mode functions are oscillatory and oscillate at similar wavelengths as the peak wavelength of the measured wave field and thus have clear correspondence to the physics of the measured wave field. Also note from Figure 17 that aside from the first mode, the magnitude of the spectral peak near the peak wavelength is greatest in the smallest modes and decreases and broadens as the mode number increases. This result suggests that the smallest modes are more tightly coupled to the wave field physics than higher modes.

Figure 19 shows the energy spectrum of each of the leading twenty mode functions from POD applied to spatiotemporal Doppler velocity for dataset 246 during mixed sea
conditions. The vertical dashed lines show the peak swell and wind wave wavenumbers. All of the leading modes, with the exception of mode one, show spectral peaks at or surrounding the peak wavelengths similar to the results for dataset 269. It can be concluded that under mixed sea conditions the leading POD mode functions are also oscillatory and are dominated with oscillations near both peak wavelengths of the measured wave field and thus also have clear correspondence to the physics of the measured wave field. From this result, it can be deduced that mixed seas do not significantly change the ordering of the basis functions and in a mixed sea scenario the basis functions incorporate both swell and wind-seas in the same functions.

Figure 20 shows the reconstruction of POD mode 1 from dataset 269. This mode is likely associated with the range dependent decay of the radar signal. Excluding this mode in wave field reconstruction yields a more constant signal from near to far range. For the purposes of wave field POD reconstructions, this mode is excluded.

In order to evaluate if the POD method preferentially reconstructs wave energy as opposed to non-wave contributions to the radar Doppler measurement, energy in each mode reconstruction is examined as well as energy only inside swell and wind-wave “bands”. Figure 21 shows percentage of total reconstructed energy, as well as percentage of energy within only the swell and wind-wave energy bands, in each $n$ mode reconstruction. For each $n$ mode reconstruction, modes 1 through $n$ are used for reconstruction. The PSD of velocity is calculated for each POD mode velocity reconstruction and integrated twice to compute energy captured by that reconstruction. The swell and wind-wave wavelength bands are based on the width of the spectral energy peaks of the original Doppler velocity data associated with the swell and wind-waves for each dataset. Specifically, the swell
energy wavelength band is defined as wavelengths from 87 m to 210 m, and the wind-wave band is defined as wavelengths from 20 m to 70 m. The vertical axis in Figure 21 shows percentage of total energy \( (T_e) \), which is defined as:

\[
T_e = 100 \left( \frac{P_r}{P_o} \right)
\]

where \( P_r \) is the energy in the POD reconstructed spectrum (using modes 1 to \( n \)), and \( P_o \) is the total energy from the measured Doppler velocity spectrum (i.e., the double integration of the 2D PSD of the measured spatiotemporal Doppler velocities). The results in this figure show that the POD mode reconstructions accumulate energy from within the swell and wind-wave wavenumber bands at a faster rate than total energy is accumulated. The reconstruction of modes 1-20 represents 70% of the swell and wind-wave energy, but only 28% of the total energy. The reconstruction of modes 1-100 represents 99% of the energy in the swell and wind wave spectral bands. This result shows that a leading mode reconstruction would contain more energy in the wave bands than energy outside of these bands, which could be associated with potential non-wave contributions to the radar measurement.

To further investigate the energy distribution of the reconstructed velocity as a function of wavenumber and mode number, \( \omega-k \) spectra for each mode reconstruction are computed. Specifically, only the time series coefficients and one basis function is used to reconstruct the velocity, where the coefficients and basis function used changes from that associated with mode 1 to 2048 (maximum mode for this dataset). This \( \omega-k \) spectrum is subsequently integrated over \( \omega \) to yield a 1D wavenumber spectrum for each mode velocity reconstruction. This 1D wavenumber spectrum (for each mode reconstruction) is
normalized by the integrated (over $\omega$) $\omega-k$ spectrum of the measured spatiotemporal Doppler velocity data. Thus, the values of the normalized energy spectra represent the percentage of measured energy at each wavenumber contained in each mode velocity reconstruction, as shown in Figure 22.

Panels A and B of Figure 22 represent run 246, the bi-modal system consisting of swell and wind-waves, while panels C and D represent the wind-wave dominated run 269. Panels A and C show the complete POD mode range of each dataset, from mode 1 to 2048, while panels B and D show a zoom-in of panels A and C that encapsulate modes 1-25 for each dataset. The white dotted lines represent spectral energy peaks ($\lambda_p$ for swell and wind-waves) as measured by the wave buoy. Similar to the results presented in Figure 21, no significant amount of energy in the swell and wind-wave wavelength bands is present in modes larger than 100. Figures 21 and 22 also show that the amount of energy above mode 100 is mostly less than 1% of the original energy aside from some short wavelength (high frequency) energy contained in modes larger than 100. This result indicates that energy content in modes greater than 100 is spread over many modes with no concentrated energy in any individual mode at any wavelength.

Panel B of Figure 22, representing run 246, shows reconstructions based-on mode 2, 3 and 4 contain energy predominately concentrated at the peak swell wavenumber and reconstructions based-on mode 5, 6, and 7 contain energy predominately concentrated at the peak wind-wave wavenumber. Reconstructions based-on mode 8 through mode 20 contain energy at wavenumbers surrounding the peak swell and wind-wave wavelengths, which can act to broaden the spectral peaks around the peak wavenumbers. Above mode
20, limited swell and wind-wave energy is contained in the individual mode velocity reconstructions.

Panel D of Figure 22, representing run 269, shows velocity reconstructions based-on mode 2 through mode 8 contain energy near the peak wind-wave wavenumber. Reconstructions based-on mode 8 through mode 20 predominantly adds energy surrounding the peak wavenumber, broadening the spectral peak. As in run 246, above mode 20 no significant wind-wave energy is contained in individual mode reconstructions. Above mode 100, short wavelength (high frequency) energy is contained in the modes, but as seen in the lower left panel (C), little energy is added by any individual mode at any consistent wavelength (at modes above 100).

In summary, these results demonstrate that the leading modes (aside from the first mode) are associated with energy near the peak wavelengths regardless of system modality and that the modes have a correspondence with the physics of the wave field, where specifically, the oscillatory mode shapes have wavelengths representative of the wave field. Collectively, these results suggest that using the modes as a filter to retain wave field energy preferentially over other sources of variance in the Doppler signal is feasible because the leading order modes can be associated with the wave field physics.

5.1.2. Rotating Radar

In this section, the mode basis functions of synthetic and measured rotating radar data are examined to determine if the mode basis functions correspond to the physics of the measured wave field. As previously discussed, the rotating radar data differs from the staring radar data because the Doppler velocity is a function of two spatial coordinates (i.e.,
\( \mathbf{D}(x,y) \) rather than range and time (i.e., \( \mathbf{D}(r,t) \)) and therefore it cannot be assumed \textit{a priori} that the basis functions derived from the POD method will be the same for the rotating and staring radars. However, as described in the Methods section, the Doppler data are first rotated so that the predominant wave propagation is along the \( x \)-direction, which is the same direction that the mode functions are associated. Due to this rotation, similar behavior as observed with the starting radar is expected, but the results presented in this section demonstrate the similarity.

Figure 23 shows the first mode basis function of POD applied to a frame of emulated rotating radar Doppler velocity data (emulator run 1). This figure shows that the mode function is oscillatory and Figure 24 shows the spectrum of this mode where it can be seen that the mode’s spectral peaks are consistent with the peak wavelengths from the directional wave spectrum from which the emulator run was initiated (see Figure 12; section 3).

Figure 25 shows the 1D wavenumber spectrum for each individual mode reconstruction for POD applied to a frame of Doppler velocity data measured using the rotating UM radar. The 2D \( k_x-k_y \) spectrum is calculated from the individual mode velocity reconstructions and integrated over \( k_y \) to yield a 1D wavenumber spectrum for each mode reconstruction (between 1 to 141 for this dataset). Each 1D wavenumber spectrum is then compiled to form the spectrogram show in this Figure (25). The vertical dashed white line represents the peak wavelength as measured by the GPS wave buoy during the time when the dataset was collected. Energy contained in the modes 1 through 10 velocity reconstructions are largest around the peak wavelength. Above mode 10 reconstruction, small amounts of energy are spread across the remaining reconstructions/modes and across many
wavelengths. This energy is presumably part of non-wave contributions to the radar Doppler velocity measurement. Recall that the UM rotating radar range resolution is much coarser than the staring radar (see Table 1, section 3), which results in more noise in the Doppler data.

From these results, we can deduce that the POD performs similarly for the rotating radar as the staring radar in that the leading POD mode functions are oscillatory and have wavelengths representative of the measured wave field.

5.2. Comparison of POD and FFT Methods

In this section POD wave field reconstructions will be compared to traditional dispersion curve filtering Fourier based methods. Wave statistics from both methods are compared to “ground truth” buoy calculated statistics (as described in section 4.4.), statistical dependencies on environmental conditions are examined, and phase-resolved velocity reconstructions are compared to buoy measured wave orbital velocity records.

5.2.1. Statistical Comparison

In this study, the wave field is characterized by three commonly used statistics: significant wave height ($H_s$), peak wavelength ($\lambda_p$), and root mean squared orbital velocity ($V_{rms}$). These statistics are calculated for the best wave field reconstruction using POD and dispersion filtering techniques. The best reconstruction for each method is defined as the reconstruction with the smallest $H_s$ error. Dispersion curve filter widths of $1\Delta\omega$ to $10\Delta\omega$ are investigated as well as all possible leading mode reconstructions (from mode 1 to mode
$n$) for the POD method are examined, to determine the best reconstruction for each method for comparison.

Figure 26 shows the number of modes needed for the best mode reconstruction (1 through $n$) versus the buoy measured $H_s$ for all rotating radar datasets examined (see Table 3; section 3). The UM radar is marked in black dots and the APS radar in magenta crosses. The best mode shows a weak trend with increasing significant wave height; however, the majority of datasets are accurately reconstructed by fewer than the leading 20 modes (approximately 14% of the total modes). This low number of modes required for an accurate reconstruction could reduce storage demands for large datasets. As wave height increases the complexity of the wave field increases, potentially slightly increasing the number of required modes to accurately reconstruct the wave field.

Figure 27 shows the best dispersion curve filter width versus buoy measured $H_s$. Typically a filter width of $1\Delta\omega$ to $3\Delta\omega$ visually encompasses the majority of energy associated with the linear dispersion curve. However this figure shows that for the majority of datasets examined, the most accurate reconstruction was greater than $3\Delta\omega$, and often at or near the upper limit of filter widths examined. This result implies that some of the energy off of the linear dispersion curve is needed to obtain accurate $H_s$ statistics when the $H_s$ statistic is computed from 1D spectra along the peak wave propagation direction. Use of such a large dispersion curve filter width is unlikely to be chosen \textit{a priori}.

Figure 28 shows histograms of the error of the peak wavelength of the wave field reconstructions. Peak wavelengths are compared using the buoy identified dominant wave system when two wave systems were present. In such cases (see Table 3), particularly when
the two systems were of similar magnitude, the buoy often measured the higher energy on the wind wave peak while the radar the swell peak. This discrepancy is likely due to the fact that the radar revisit rate to the same ocean patch results in a maximum observable frequency (Nyquist frequency) near the wind wave frequency peak, particularly when the ship had a non-negligible forward speed into the wave propagation direction. Thus, the radar is more sensitive to the swell, while the buoy is small in size (~0.5 m diameter) and is therefore sensitive to the wind seas. The blue bars are dispersion filtered datasets and the orange bars are POD datasets. Panel A (left) shows the APS radar datasets and panel B (right) shows the UM datasets. This figure shows that the majority of all reconstructions for both methods and radar systems are within less than 10% error in buoy-identified \( \lambda_p \), and all datasets are with 25% error. This result confirms that both reconstruction methods are accurately capturing the buoy-identified dominant wave system. The UM radar POD results are generally more normally distributed with a smaller spread than the APS radar POD results. The POD datasets generally show higher frequency of lower errors than the dispersion filtered datasets for both radars.

Figure 29 shows histograms of the error of POD and dispersion filtered \( V_{rms} \) estimations with buoy measured \( V_{rms} \). POD estimates are slightly lower in error than dispersion filtered estimates. The discrepancy between buoy measurements and radar estimations for \( V_{rms} \) is most likely caused by the buoy being more sensitive to higher frequency wind waves than the radar measurements. As discussed in the prior paragraph, high frequency wind waves are close to the high frequency limit of the radar measurement, particularly when the ship is moving into the direction of propagation of the waves. In addition, the buoy measures the entire orbital velocity of both wind waves and swell, where as any wave system
component not aligned with the peak direction is underestimated by the radar because it only measures a projection of that velocity component.

Figure 30 shows the buoy, POD, and dispersion filtered estimated $H_s$ with error bars calculated as described in section 4.3. Panel A (left) shows APS datasets and panel B (right) shows UM datasets. For all datasets but the lowest $H_s$ for the APS radar and the lowest three $H_s$ for the UM radar, the POD $H_s$ estimate falls within the error of the buoy measurement. The dispersion filtering method fails to accurately measure the significant wave height for the same datasets as the POD method and underestimates the significant wave height for an additional dataset for the UM radar. Overall the POD and dispersion filtering methods perform comparably in terms of accurately estimating significant wave height, but the POD method estimates $H_s$ accurately for one additional dataset.

Relationships of statistical reconstruction accuracy (in terms of $H_s$, $V_{rms}$, and $\lambda_p$) are examined with respect to various environmental conditions (Figure 31). The environmental conditions examined are $U_w$, $H_s$, $\lambda_p$ (for swell and wind waves), $V_{rms}$, and the directional separation between swell and wind wave systems ($\Delta \Theta$). However, no obvious dependency of reconstruction accuracy with any specific environmental condition examined is identified.

In summary, POD and dispersion filtered reconstructions are statistically comparable. Both methods accurately capture the buoy-identified dominant wavelength of the ocean wave field, while the POD method and the UM radar generally show the highest frequency of low $\lambda_p$ errors. While both methods accurately estimate the buoy measured $H_s$ for the majority of datasets examined, the dispersion filtering method fails to fall within the error
margins of the buoy measurement one more time than the POD method. Finally, no clear relationship between the environmental conditions examined and the wave statistic’s accuracy is evident; suggesting that the POD method’s results do not depend strongly on the environmental conditions of the Doppler radar data collection. In addition, results for both radars were similar suggesting that the POD method’s results also do not depend strongly on radar configuration.

5.2.2. Phase-Resolved Comparison

In order to characterize the phase accuracy of the wave orbital velocity reconstructions, comparisons to buoy time series are necessary. Radar reconstructed wave fields are geolocated and velocity time series are calculated as described in section 4.5. Figure 32 shows an example frame of UM Doppler velocity data (dataset 7; Table 3, section 3) and the overlaid buoy tracks within the radar field of view. Pearson’s correlation coefficient (C; Equation 28) and root mean squared error (Erms; Equation 29) are calculated between buoy velocity time series and the corresponding POD and dispersion filtered time series for each available instance when a GPS wave buoy was within the field of view of the radar. The correlation and Erms statistics are calculated individually for each buoy-radar time series pair for dispersion filter widths from 1Δω to 10Δω and for all leading mode reconstructions, then averaged between all available buoys. While C and Erms are shown for all leading POD mode reconstructions, only the C and Erms for the filter width with the highest C is shown.

Figures 33 and 34 show C and Erms respectively for UM dataset 7 with four available buoy time series for comparison. The POD reconstruction C is significantly higher than the best
dispersion filter width correlation for all POD mode reconstructions for this dataset. The POD \( E_{rms} \) is slightly lower than that of the best dispersion filter width for all mode reconstructions, but they are comparable in magnitude.

Figures 35 and 36 show \( C \) and \( E_{rms} \) respectively for APS dataset 7, with four available buoy time series for comparison. The POD reconstruction \( C \) is significantly higher than the best dispersion filter width correlation for all POD mode reconstructions for this dataset, similar to the results for the same time frame for the UM radar. The \( E_{rms} \) for the dispersion filtering method is significantly lower than that of the POD method for all mode reconstructions.

Finally, Figures 37 and 38 show \( C \) and \( E_{rms} \) respectively for the same UM dataset 8. The highest POD mode reconstruction \( C \) is equal to that of the best dispersion filter width, while the POD \( E_{rms} \) is lower for most mode reconstructions than for the best dispersion filter width but they are similar in magnitude.

Discrepancies between the correlation results (\( C \)) and RMS velocity errors (\( E_{rms} \)) are attributed to errors resulting from phasing versus amplitude. In other words, correlation is high when the phasing of the waves is coherent between the two time series, while magnitudes of the orbital velocities are more accurate when \( E_{rms} \) is small. Thus, a high correlation and high \( E_{rms} \) implies that the phasing is accurate but the magnitude of the orbital velocities is either over or underestimated.

5.3. Discussion

Analysis of the POD mode function shapes for both rotating radar systems and the staring radar show that the mode functions are oscillatory and contain energy at wavelengths
representative of the measured wave field. These findings are consistent with the application of POD to the perfectly (i.e., no noise or radar scattering artifacts) measured single and multi-frequency sinusoidal wave fields as well as to Bretschneider spectrally modeled wave fields in Hackett et al. (2014). This study found that in all cases, the leading POD modes were oscillatory, contained energy at wavelengths representative of the measured wave field, and accurately reconstructed the modeled wave field. It was also shown that the presence of two wave systems does not adversely affect the POD method.

Statistical comparison shows that the two methods perform similarly across the range of environmental conditions and radar systems examined. Both methods accurately estimate the $H_s$ within the error of the buoy measurement for the majority of datasets examined. While no dependency of accuracy on any environmental condition was found, a weak trend of increasing dispersion filter width and best POD mode with increasing $H_s$ was evident. When wave frequency is held constant, increasing $H_s$ will increase orbital velocity and wave steepness, and as a result increase wave breaking. This would increase the amount of group line energy and complexity of the $k$-$\omega$ spectrum, thus making a wider dispersion filter width preferable in order to include some non-dispersion curve associated energy, and more modes are also needed to account for the complexity of the wave field.

Figure 39, panels A, B, and C show the $k$-$\omega$ spectrum associated with the phase-resolved analysis results presented in section 5.2.2 (UM datasets 7 and 8 and APS dataset 7). All spectra have a comparable energy range, and were collected under similar $H_s$ conditions (1.6 and 1.7 m). However, UM dataset 8 (Figure 39, panel A) shows a greater amount of energy lying along the linear dispersion relationship and a (relatively) lower energy, less defined group line compared to dataset 7 (Figure 39 panels B and C). For UM dataset 8,
with a greater amount of dispersion curve energy and weaker group line, Figure 37 shows that the dispersion filtering method attains a slightly higher $C$ than the POD reconstruction method. However for both radars, dataset 7, with a stronger group line feature, Figures 33 and 34 show the POD method attaining a significantly higher $C$ than the dispersion filtering method. This discrepancy in correlation is most likely attributed to the strength of the group line energy relative to the strength of the dispersion curve energy, and the importance of non-dispersion curve wave energy to the accuracy of the reconstructed wave field.

The lower row of panels (D, E, F) of Figure 39, show the $k$-$\omega$ spectrum for the POD reconstructions that yield the most accurate $H_s$ statistic for each dataset. Note that for each POD reconstruction energy spectra, energy both on the linear dispersion curve (dotted white line) and energy associated with the group line feature is present. These results suggest that the group line energy included in the POD method improves the phasing accuracy of the orbital velocity time series despite both methods generating equivalent statistical representations of $H_s$ (see Figure 30).

Further support of this conclusion is provided by the emulated rotating radar results. Figure 40, panel A shows a comparison of average correlation between POD or dispersion filtered generated orbital velocity time series and simulated radial orbital velocity time series for the radar emulator simulation with no group line present in the $k$-$\omega$ spectrum (Figure 40, panel B), and for the simulation with a group line present (emulator run 1; Figure 40, panel C). In this figure, the correlation between the simulated radial orbital velocity and that extracted from the simulated Doppler radar measurements for both methods is computed for each range bin and those correlation coefficients are subsequently averaged over all range bins to generate an average correlation coefficient. Presence of the group line
negatively affects the average correlation of the dispersion filtering method, which attains a higher correlation than the POD method for the non-group line emulator results (emulator run 2). The dispersion-filtered results for the non-group line simulation (emulator run 2) yield the highest average correlation using a narrow dispersion filter width ($1\Delta\omega$). However when the group line is added, the highest average correlation dispersion filter width increases from $1\Delta\omega$ to $10\Delta\omega$. Expansion of the dispersion filter width is indicative that energy off the dispersion curve is required for an accurate orbital velocity reconstruction when group line energy is present in the $k$-$\omega$ spectrum. When the group line is present, the POD method attains a higher average correlation than the dispersion filtering method. Correlation of the POD method increases until approximately 40 modes (1-40 mode reconstruction), where it plateaus. The presence of group line energy in the $k$-$\omega$ spectrum of the 1-40 mode reconstruction (Figure 40 panel D) further indicates the importance of the inclusion of group line energy to an accurate orbital velocity reconstruction.

Plant and Farquharson (2012) concluded from numerical simulations that group line energy observed in $k$-$\omega$ spectra is related to interference-induced wave breaking. This wave breaking is caused by the interaction of short wavelength surface gravity waves with ocean currents or longer wavelength surface waves. Irisov and Voronovich (2011) also showed in a numerical study that short wavelength ocean surface waves break due to local steepness maxima or currents. It has been shown that wave breaking contributes to radar Doppler velocity measurements of the ocean surface (Hwang et al., 2008).

The analysis in this study provides experimental evidence that incorporation of energy associated with the group line yields improved correlation between orbital velocity time series when significant group line energy is present; thus, capturing the effects of this wave-
wave or wave-current interference improves phase-resolved time series comparisons. The POD method is able to capture this group line energy while dispersion filtering neglects it. Consequently, there are advantages to the POD method when accurate wave phasing is necessary. Conversely, the energy on the dispersion curve seems to impact most directly $E_{rms}$ and the POD method does not isolate the energy on the dispersion curve causing the dispersion filtering method to outperform the POD method in a statistical sense (i.e., $E_{rms}$) over short time periods (~30 s). However, the statistical results previously shown indicate that over a larger period of time and space this effect is diminished because when examining statistics over several minutes (rather than ~30 s) the wave field statistics from both methods are comparable.
Figure 17: Power spectral densities (PSD) of the first twenty POD mode basis functions for dataset 269 (wind seas only). The dashed vertical line show the buoy measured peak wind-wave wavenumber.
Figure 18: POD mode function two for dataset 269 (wind seas only).
Figure 19: PSD of the first twenty POD mode basis functions for dataset 246. The dashed vertical lines show the buoy measured peak swell and wind wave wavenumbers.
Figure 20: Velocity reconstruction using only POD mode one. This mode is likely associated with the range dependent radar signal decay.
Figure 21: Normalized cumulative energy ($T_e$) versus $n$ mode reconstruction, and normalized cumulative energy within swell and wind wave (WW) wavelength bands. Swell and wind-wave energy is accumulated at a faster rate than total energy.
Figure 22: 1D normalized wavenumber spectra of individual mode reconstructions versus mode number. Panels A and B correspond to run 246. Panels C and D correspond to run 269. The right panels show a zoomed-in view of the left panels.
Figure 23: First mode function from POD applied to a single frame of emulated rotating radar Doppler velocity measurements (emulator run 1).
Figure 24: Wavenumber spectrum of first mode function of POD applied to emulator run 1 (Figure 23).
Figure 25: 1D wavenumber spectra of individual mode velocity reconstructions versus mode number for rotating UM radar (dataset 14). Panel A (left) shows the full range of POD modes and panel B (right) shows a zoom-in of the leading 25 modes.
Figure 26: Most accurate mode (reconstruction of modes 1 through n) with respect to $H_s$ versus buoy measured $H_s$. 
Figure 27: Most accurate dispersion filter width ($W$; Eqn 16 with respect to $H_s$ versus buoy measured $H_s$.)
Figure 28: Histograms of the distribution of $\lambda_p$ errors of POD reconstructions (orange) versus dispersion filtered (blue) for APS radar system (panel A) and UM radar system (panel B).
Figure 29: Histograms of the distribution of $V_{rms}$ errors of POD reconstructions (orange) versus dispersion filtered (blue) for APS radar system (panel A) and UM radar system (panel B).
Figure 30: $H_s$ estimates with error bars (calculated as described in section 4.3) for radar-based POD and dispersion filtered methods along with buoy measured values. Panel A shows APS datasets, panel B shows UM datasets.
Figure 31: Scatter plots of environmental conditions versus $V_{rms}$ error. Diamonds represent POD method, open circles represent dispersion filtering method, black symbols represent UM radar system, and magenta symbols represent APS radar system. Symbol size reflects number of POD modes or dispersion filter width for most accurate $H_s$ reconstruction. Panel A shows $V_{rms}$ error vs $\lambda_p$, panel B shows $V_{rms}$ error vs $H_s$, Panel C shows $V_{rms}$ error vs $U_w$, and panel D shows $V_{rms}$ error vs $\Delta \Theta$. 
Figure 32: This figure shows an example frame from UM dataset 7 with four available buoy tracks (shown in red).
Figure 33: Correlation coefficient ($C$) between orbital velocity time series determined from radar-based dispersion filtering technique and buoy time series for the dispersion filter width (red) with the highest $C$ along with the corresponding $C$ for the radar measured POD orbital velocity reconstructions for each POD mode reconstruction (1 to $n$)(black) for UM dataset 7 (Table 3; section 3).
Figure 34: $E_{rms}$ between orbital velocity time series determined from radar-based dispersion filtering technique and buoy time series for the dispersion filter width (red) with the highest $C$ along with the corresponding $E_{rms}$ for the radar measured POD orbital velocity reconstructions for each POD mode reconstruction (1 to $n$) (black) for UM dataset 7 (Table 3; section 3).
Figure 35: Correlation coefficient ($C$) between orbital velocity time series determined from radar-based dispersion filtering technique and buoy time series for the dispersion filter width (red) with the highest $C$ along with the corresponding $C$ for the radar measured POD orbital velocity reconstructions for each POD mode reconstruction (1 to $n$)(black) for APS Dataset 7 (Table 3; section 3).
Figure 36: Correlation coefficient ($C$) between orbital velocity time series determined from radar-based dispersion filtering technique and buoy time series for the dispersion filter width (red) with the highest $C$ along with the corresponding $C$ for the radar measured POD orbital velocity reconstructions for each POD mode reconstruction (1 to $n$)(black) for APS dataset 7 (Table 3; section 3).
Figure 37: Correlation coefficient ($C$) between orbital velocity time series determined from radar-based dispersion filtering technique and buoy time series for the dispersion filter width (red) with the highest $C$ along with the corresponding $C$ for the radar measured POD orbital velocity reconstructions for each POD mode reconstruction (1 to $n$) (black) for dataset 8 (Table 3; section 3).
Figure 38: $E_{rms}$ between orbital velocity time series determined from radar-based dispersion filtering technique and buoy time series for the dispersion filter width (red) with the highest $C$ along with the corresponding $E_{rms}$ for the radar measured POD orbital velocity reconstructions for each POD mode reconstruction (1 to $n$) (black) for UM dataset 8 (Table 3; section 3).
Figure 39: Panels A, B, and C show the $k$-$\omega$ spectrum for UM dataset 8, UM dataset 7, and APS dataset 7 respectively. Panels D, E, and F show the $k$-$\omega$ spectrum for the POD reconstructions with the most accurate $H_s$. The linear dispersion relationship is shown as the white dotted line.
Figure 40: Panel A shows the averaged correlation for the emulator run with a group line present in the $k-\omega$ spectrum (blue lines) and without a group line present in the $k-\omega$ spectrum (black lines) for correlations between orbital time series generated from the dispersion filtering technique and the simulated radial orbital velocities for the optimal dispersion filter width (i.e., width that generated the largest average correlation) (dotted-dashed lines) along with the averaged correlation from the corresponding POD orbital velocity reconstructions for all 1 to $n$ mode reconstructions (solid lines). Panel B shows the radar $k-\omega$ spectrum for the emulator simulation with no group line present (emulator run 2). Panel C shows the $k-\omega$ spectrum for the emulator simulation with a group line present (emulator run 1). Panel D shows the $k-\omega$ spectrum of the POD reconstruction of the group line emulator simulation at the point where $C$ begins to plateau (40 mode). The linear dispersion relationship is shown as a white dotted line for all panels.
6.0. Summary and Conclusions

Marine Doppler radar systems can be used as a remote sensing tool to measure ocean surface velocities. Conventionally, FFT-based dispersion curve filtering is applied to Doppler velocity measurements to separate orbital velocity from non-wave contributions to the measurement (Young et al., 1985). However, non-linear wave energy is not associated with the linear dispersion relationship, and is thus eliminated by dispersion curve filtering. This research applied and evaluated an alternative method, POD, to extract ocean surface wave information from Doppler velocity measurements of the sea surface.

Results of this study show wave contributions to Doppler velocity measurements of the sea surface can be separated from non-wave contributions using a subset of leading POD mode functions. This result was insensitive to radar configuration and environmental conditions such as sea modality, angles between swell and wind waves, significant wave height, wind speed, and peak period.

Because there is no innate relationship between the mode basis functions and the physics of the measured wave field, the content of leading modes was examined. The 1D spectra of the lowest mode functions were found to contain significant variance at the peak wavelength of the wave field as measured by wave buoys. This connection between the leading POD modes and the physics of the wave field enables the use of the POD method as a filter to isolate wave contributions to the Doppler velocity measurement.
Significant wave height, RMS orbital velocity, and peak wavelength wave statistics were calculated from both POD and FFT reconstructed orbital velocity maps. Both methods were found to perform similar statistically. Buoy $H_s$ was accurately estimated by both reconstruction methods for the majority of datasets, with the exception of the lowest $H_s$ cases ($H_s < 1$ m); although, varying number of modes and dispersion filter widths were needed to optimize the match to buoy $H_s$. Dispersion filter widths of $W = 1$ to $W = 10$ were investigated (Eqn. 16), and it was found that for the majority of datasets the largest or near largest dispersion filter width was required for the most accurate $H_s$ statistic. The wide dispersion filter suggests the importance of including non-dispersion curve associated energy in the reconstruction. For the POD method, in the majority of cases examined, less than 15% of the modes were needed to obtain the optimal match to the buoy $H_s$.

To evaluate accuracy of phase-resolved orbital velocity maps, correlation coefficients between FFT or POD reconstructed orbital velocity time series and ground truth orbital velocity time series were calculated from both experimental and numerically modeled radar Doppler velocity data. It was found for the experimental data that when group line energy is high relative to dispersion curve associated energy that the POD method attains a higher correlation coefficient for all mode reconstructions. In contrast, when group line energy was not high relative to dispersion curve energy, the FFT-based dispersion curve and POD methods performed similarly with the optimal mode and dispersion filter width selection.

For the numerical data, results from two model runs were compared: one with a group line and another without a group line. Results for the run containing only dispersion curve associated energy showed the FFT method attained a higher correlation than the POD method using a $W = 1$ dispersion filter width. In contrast, for the numerical model run that
contained group line and dispersion curve energy, the highest correlation was attained using a $W = 10$ filter width, suggesting the importance of including some of the group line energy. In addition, the POD method attained higher correlation coefficients for mode reconstructions beyond $n = 20$. The 2D spectra of POD reconstructed orbital velocity maps show both dispersion curve associated energy as well as group line energy. Collectively, these results demonstrate experimentally that energy in the group line feature contains contributions from the wave field, and at least a portion of it needs to be included to obtain high correlations with ground truth orbital velocity time series (when the group line energy is large relative to the energy on the linear dispersion relationship). This result supports the findings of Plant and Farquharson (2012) who found numerically that one of the contributors to group line energy is wave interference induced breaking.

This research into an alternative signal processing method for the extraction of wave orbital velocity information from radar Doppler velocity measurements of the sea surface contributes toward the development and advancement of real-time sea state awareness and forecasting technologies. The application of this alternative method could improve methodologies associated with the generation of phase-resolved sea surface elevation maps, which in-turn can be used to improve safety and increase operational awareness in commercial and defense applications, as well as provide vital information for increasing knowledge in a number of oceanographic research areas (such as wave evolution/growth, rogue waves, wave-wave energy exchange, and wave-current interaction).
7.0. References


