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Engaging Students with High-stakes Problems

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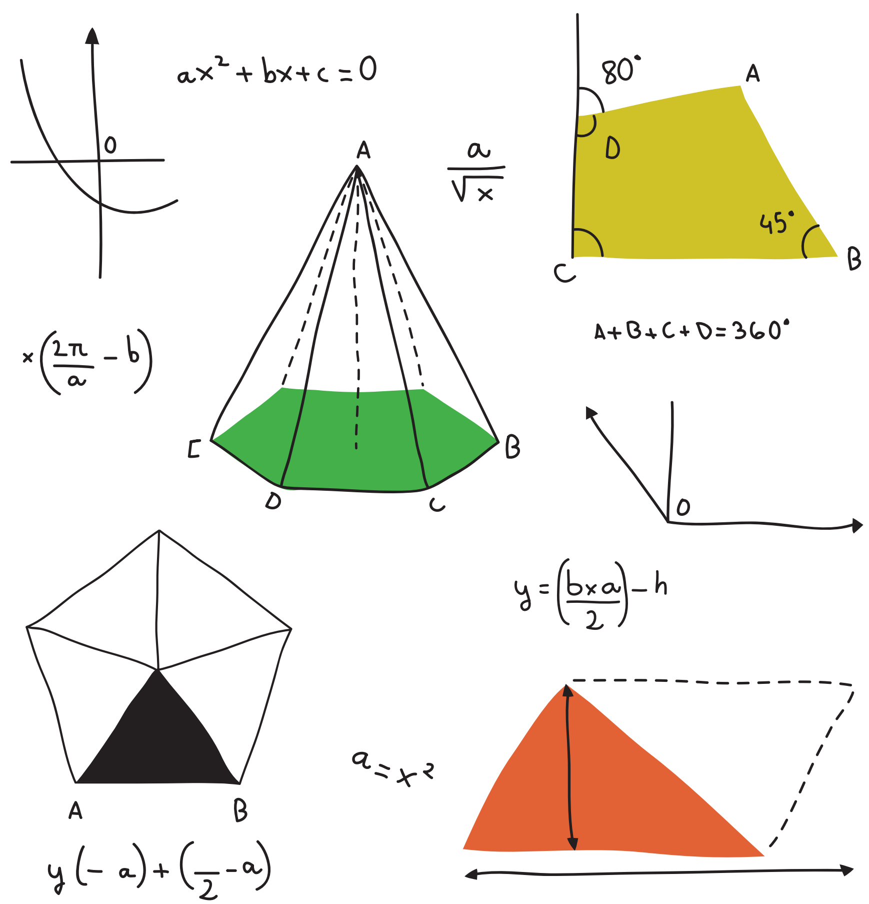

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Engaging students with high-stakes problems
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Engaging students in meaningful mathematics problem-solving is the intention of many education stakeholders around the world. Research suggests that the implementation of high-stakes problems in mathematics teaching is one way to strengthen students' conceptual understanding. Many carefully crafted open-ended problems constitute high-stakes problems, and proper use of such problems in teaching and learning not only encourages learners' flexible thinking but also helps detect their misconceptions. However, what is less practiced and understood is: how exactly one should aim to implement such problems in a classroom setting. Teaching pre-service middle school teachers for a few years using high-stakes (mostly open-ended problems) has given me insights that may be useful to teachers around the world. In this paper, I share my experience of teaching with high-stakes problems. We will demonstrate how user-friendly interactive graphing tools can be used in the creative process of problem-solving.

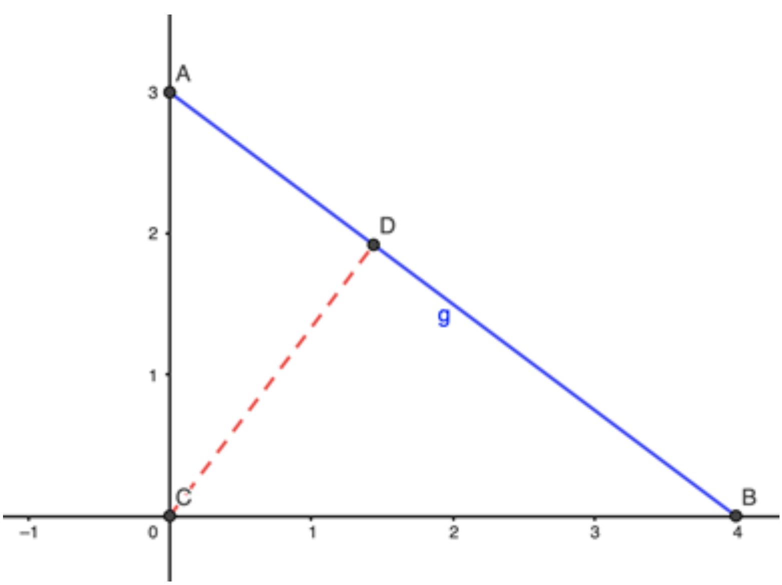
Introduction

A mathematical problem refers to any circumstance that requires the use of mathematical tools and concepts to find a solution. To comprehend mathematical concepts thoroughly and utilize them in practical ways, students must undergo the process of problem-solving. Students learn to solve problem in mathematics primarily through 'doing, talking, reflecting, discussing, observing, investigating, listening, and reasoning' (Copley 2000, p. 29). Problems that allow for various solution methods and the meaningful application of algebraic notations, along with the use of visual aids, can contribute significantly in mathematical problem-solving (Woodward et al., 2012). Studies show that open-ended problems which have multiple correct answers possible have the potential to develop fluency, flexibility, and originality in students' problem-solving skills (Kwon et al, 2006). In this regard, Johnson and Kaplinsky (n.d) created mathematics problems known as "open middle problem" to enhance students' problem-solving abilities and critical thinking skills, and typically allows for multiple approaches to reach a solution. These types of problems may seem uncomplicated and procedural at first but can be more difficult and intricate than they appear. They offer several ways of solving them, with the objective of obtaining the best or optimal answer. Klerlein and Hervey (2019) propose that effective problems are those that are both accessible and extendable, enable individuals to establish connections, foster discussion, and communication, encourage originality and innovation, stimulate questions such as "what if?" and "what if not?", and incorporate an element of surprise.

The purpose of this paper is to discuss the classroom implementation of high-stakes problems that are relevant to middle and high school. Our aim is to present a set of 'open middle problems' and provide guidance on incorporating them into the middle school classroom. To solve these problems, students must establish links between mathematical concepts, apply existing knowledge to novel situations, and create and uncover mathematical tools in their unique manner. We use visual aids during this process to help students actively participate in constructing solutions.

Problem 1: The line CD passes through the origin is perpendicular to line AB. Find the length of the line CD.

Figure 1: Finding the length of the perpendicular line using various techniques.



Inspired by Nerdypoo (n.d), the problem above, although not an open-ended problem, can be solved in many ways. This problem allows students to explore different mathematical concepts that are relevant in middle school and beyond. No pathways are suggested, and students are forced to devise their own plans to get to the solution. We now present the solution to the problem in four different ways:

Using the area of triangle

First, we find the area of the triangle using the CB as a base and AC as a height. So that Area $\Delta ABC=1/2\ CB*AC=1/2\ 3*4=6$. Then, we find hypotenuse AB using the pythagorean theorem, $AB^2=AC^2+CB^2$. Putting the known values and taking the positive square root, we find $AB=\sqrt{3^2+4^2}=5$. Now, using the area of triangle again, but this time, we take AB as a base and CD as a height, we get,

Area $\Delta ABC=1/2\ AB*CD$. As we already know the area and base of the triangle, we now, solve for CD. So, $6=1/2\ 5*CD$. Therefore, $CD=12/5$.

Using similar triangles

ΔBAC and ΔBCD are similar triangles because angle A and C are right triangles, and both share the angle B. Therefore, $BA/AC=BC/CD$. $5/3=4/CD$, $CD=12/5$.

Finding the coordinate of D and using distance formula

We can view the point D as the point of intersection of two lines. First, we can find the equation of lines passing through the point (0,3), and (4,0). By using the slope formula $(y_2-y_1)/(x_2-x_1)$, we get the slope to be -¾. Then using the y-intercept (0,3), we find the equation of the line to be $y=-3/4\ x+3$. The perpendicular line will have the slope 4/3, and it passes through the origin. Therefore, the equation of the line become $y=4/3\ x$. Now, setting them equal, we get $-3/4\ x+3=4/3\ x$. Now solving this equation, we get $x=36/25$. Putting this value in $y=4/3\ x$ gives $y=48/25$. Now using the distance formula between two points (0,0), and (36/25,48/25) gives $CD=\sqrt{(36/25)^2+(48/25)^2}=4/25\ \sqrt{9^2+12^2}=4/25\ \sqrt{81+144}=4/25\ \sqrt{225}=4/25\ 15=60/25=12/5$.

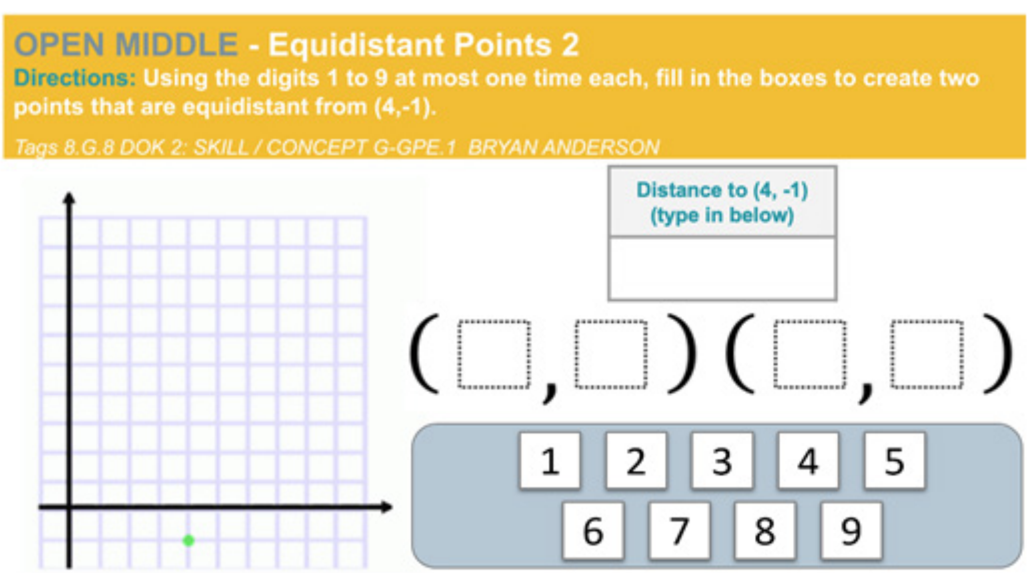
Using trigonometry

In ΔACB , we have $SinB=AC/AB=3/5$. Therefore, $B=ArcSin(3/5)$. Now, in ΔCDB, $SinB=CD/CB=CD/4$. Thus, $CD=4SinB=4Sin(arcSin(3/5))=4*3/5=12/5$.

Even though the last solution method may be a little too far to introduce in middle school, we can certainly introduce the topic for high school students. Moreover, the problem can be solved using integrals in the calculus classroom. This method is probably not appropriate to the intended audience of the paper. We ask interested readers to try finding the solution using arc length formula.

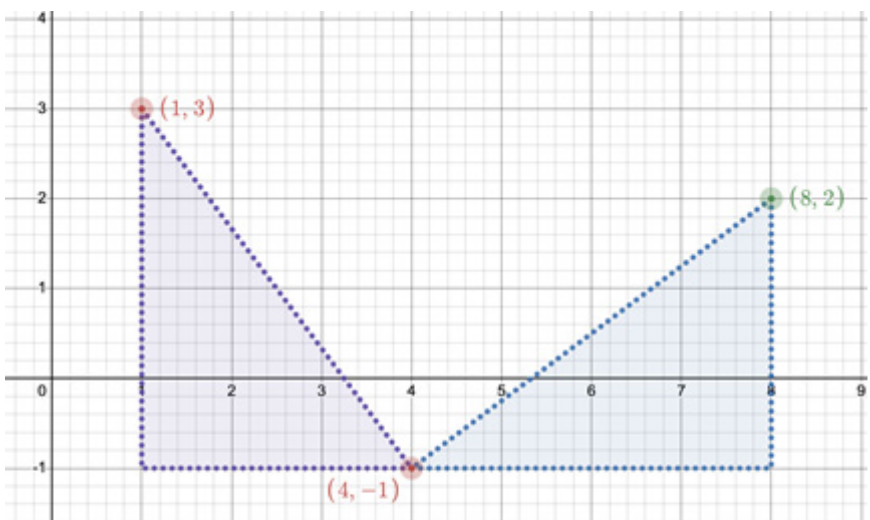
Problem 2: Using the digits 1 to 9 at most one time each, fill in the boxes to create two points that are equidistant from (4,-1). (Source: Robert Kaplinsky-Open middle mathematics)

Figure 2: Open middle problem on equidistant points



Finding equidistant points would be easier if there are no restrictions. The easier solutions can be found on horizontal and vertical lines. For example (4,0) and (4, -2) would be at equal distances from (4,-1), and there is no need to use the distance formula. Moreover, (3,-1) and (5, -1) are also at equal distances from (4, -1). However, the restriction on digits 1 to 9 will add challenges to solving this problem. In turn, students are forced to find a strategy that will help them find all different possible solutions even with these restrictions. One way to find the solution is to use the distance formula in conjunction with the guess and check method. However, doing the guess and check work with distance formulas demands too much work. We can reduce the hand calculation of the distance between two points by using the distance formula in the Desmos calculator with the use of variable points (a, b). However, soon we will realize that this will not be an effective method of solving this problem. Therefore, one needs to think strategically to find the solutions to the problem. If we can visualize the distance we are looking for it will make a hypotenuse of the right triangle, there is not even a need for calculating the actual distances. For example, one could try to form a right triangle starting at point (4,-1) with base 3 and height 4. That means the two legs of this right triangle will have lengths of 3 and 4 units so that the newly found point will be located 5 units away from (4,-1). In the picture below, the right triangle on the left side of (4,-1) would have two other vertices at (1,-1), and (1,3), and on the right will be at (7,-1), and (7,3). Both points (1,3) and (7,3) are in equal distance from (4,-1). However, this is still not a valid solution because we are repeating 3. To remedy this situation, we could create the right triangle on the right side, by going 4 units horizontally, and 3 units vertically. So the two other vertices are (8,-1), (8,2). We have found the first set of points (1,3) and (8,2) that are in equal distance from (4,-1). We can find three more sets of points by following similar reasoning. More solutions are available in the Desmos calculator at the following link https://www.desmos.com/calculator/ks41sfqxpm

Figure 3: Desmos online calculator solution to problem 2



More problems to ponder (Taken from Open Middle Problems)

Figure 4: Fractions and Decimals

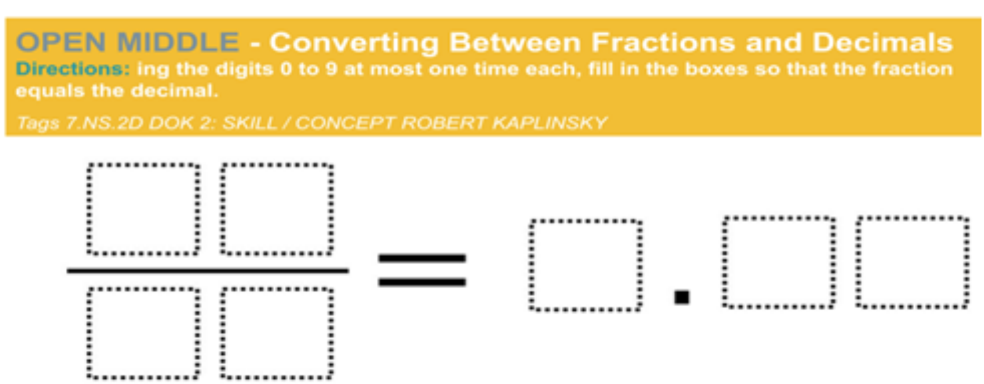


Figure 5: Minimizing Slope

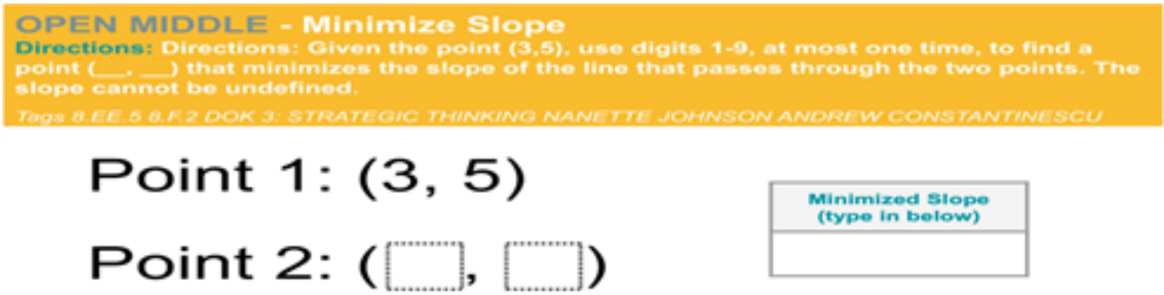


Figure 6: Comparing functions

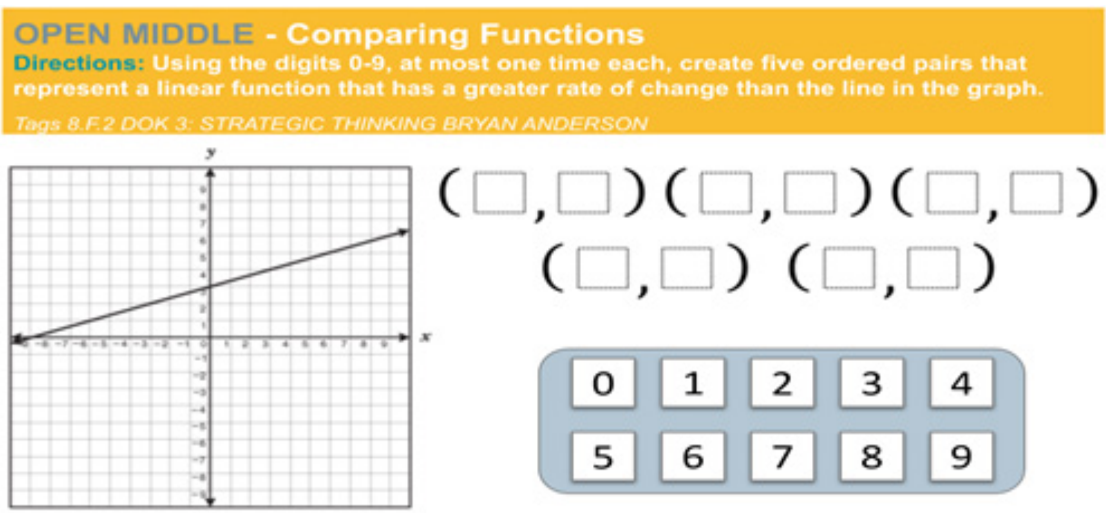
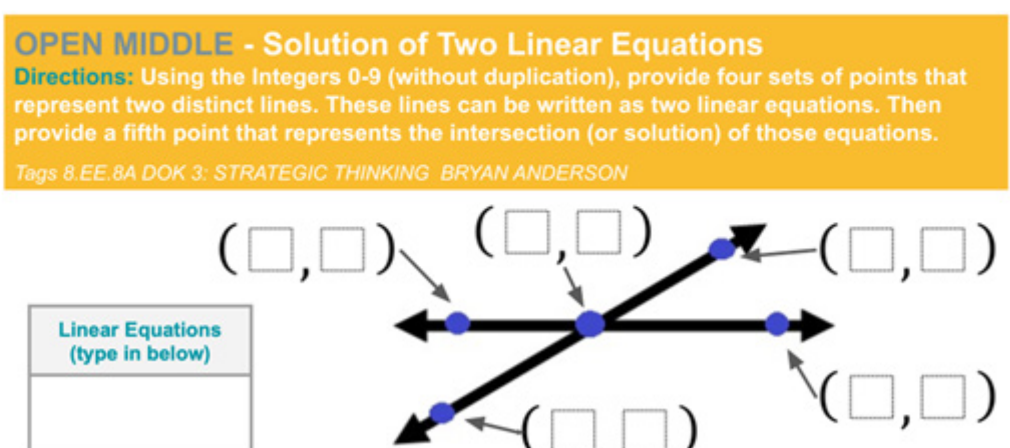


Figure 7: Solution of Two Linear Equations



Conclusions

Based on the discussion of the problems above, it is evident that the role of teachers is paramount in creating a classroom in which high-stakes problems are implemented. In a classroom like this, the responsibilities of teachers differ significantly from those in a conventional lecture-based classroom. The intention of open-ended problems is to encourage productive struggle, build a learning community, provide immediate but balanced feedback, facilitate idea sharing, and manage classroom discussion. The challenge starts with selection of the problems, setting the classroom environment, and successfully implementing it. Learning to implement effective problem-solving in the classroom is an iterative process, and it should be practiced on a regular basis to master the skill. Teachers should not limit themselves to teaching only computational skills but must strive to offer classroom experience that would be useful in dealing with real issues in students' future. Therefore, it is crucial to motivate students to engage in critical thinking and meaningful reasoning, which can have practical applications beyond the classroom and the academic sphere.

Educators around the world are invested to create readily available classroom resources. Many resources of similar nature are available on numerous websites. Some of these problems are available in GeoGebra and can be imported to the GeoGebra classroom and Google Classroom. We could even see students' progress in real-time and help or challenge them as needed. This allows a greater opportunity to differentiate the problems among students of different preparation levels.

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